

The Restless Mind 1: Math Anxiety and Math Therapy

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Beginning as I Intend to Continue

Welcome to The Restless Mind newsletter. This is, more-or-less, a continuation of The Coffee Underachiever newsletter, which I distributed via TinyLetter from 2019 to 2023.

TinyLetter shut down last year. For some time now, I have been considering how to continue. I never really loved distributing the newsletters with TinyLetter. There were technical issues trying to get my HTML to look the way I wanted it to. A couple of times, their automated bots flagged my newsletters as containing harmful content and held them back from distribution. Each time I was able to get someone on staff to review the content and release the issues, but this left me with the feeling that TinyLetter was not really the kind of distribution mechanism I wanted to use.

One possibility I wanted to explore was continuing what I was doing with The Coffee Underachiever newsletter, but using an old-school mailing list. Those of you with gray hair like me might remember these as Listservs and remember when many online (and real-world) communities grew around this kind of mailing list. But it seems that era has passed. The original LISTSERV software is now a commercial product. There is a free version, but I don't have any confidence that I could get it working on my DreamHost site. And for what I want to do with web typography, e-mail lists probably aren't the best choice.

So, my plan now is just to publish the issues on my web site, and announce the new issues via e-mail and social media, somehow — I'll leave the details to be determined later, as I am currently burned out on trying to figure out how to set up a mailing list.

Trials and Tribulations: DreamHost's Announcement Lists

I've been testing the options provided by our long-time web site and e-mail hosting company, DreamHost. It seemed like setting up a simple announcement-only mailing list that behaved like other mailing lists you are probably familiar

with ought to be easy. I wanted it to use the *de facto* standard “List-unsubscribe” header, which is supported by most major e-mail readers now, which provides readers with a one-click option to unsubscribe. I wanted to include in the footer at the end of each message a one-click unsubscribe button or link. Their online documentation showed examples of how to set this up, but I could not get the examples to work.

So, I reached out to tech support. It turns out that since I last needed tech support from DreamHost, they have severely enshittified their tech support. it seems to now be almost entirely outsourced, probably to an overseas service bureau, with replies generated mostly by bots using hallucinating AI large language models. I’d get replies referring to things in my requests that simply weren’t there, or recommendations that contradicted earlier messages. After about fifteen rounds of e-mail replies that were getting progressively worse, I called them out on Twitter:

@DreamHostCare I have been trying to get help with an issue with announcement mailing lists for a couple of weeks now. I’m on something like round 15 of e-mails with tech support. They are not answering my questions and seem like hallucinating AIs, unable to read what I wrote.

I’ve literally gotten the exact same advice multiple times in a row now, I’ve shared the error messages and the code I’m using, which exactly matches the code they recommend that I use, but they seem unable to understand what I’ve clearly documented.

It seems like every time I get a reply the tech support person says it is fixed and must have closed the issue, because the next time I reply I get a response from a new name, and the new support person hasn’t read any of the previous messages to understand the problem.

Each time, the next person comes to it “tabula rasa,” looks up the official answer, and tells me the problem will be fixed if I do the thing that I’ve been told to do now six times, AND WHICH I’VE DONE AND DOCUMENTED REPEATEDLY.

Are your tech support staff entirely LLMs, are they humans using LLMs to generate answers, or what? Because this is a disastrous waste of my time. I’ve been a DreamHost customer for [checks watch] 22 years now.

This is quite honestly the worst tech support I’ve seen in my entire career as a software engineer, a career which has now lasted for 34 years. It’s like I’m being deliberately gaslit. Can I please speak to a person?

Lo and behold, after that, I was able to have a conversation with a manager. Said manager read over the history of my requests to tech support and the responses I was getting, tested the problematic code himself, found that I was right and that said functionality was busted, apologized profusely, and gave me three months of free hosting.

The executive summary is this: the examples in Dreamhost's documentation, showing how to include an unsubscribe link or button in outgoing messages, simply do not work, and I'm told this can't be fixed and won't be fixed.

The manager went on to suggest that I use Google Workspace.

Which... I'm not going to do.

Many Bothans died to bring you this information. Also, a substantial number of my limited remaining brain cells.

Continuing with What, Exactly?

So, the newsletter. I've been feeling stuck, with a folder full of fragments, unfinished essays I've gotten bored with, and fragments of essays that never got off the ground, wondering what sort of writing project I want to work on next.

But I'm determined to continue.

A few weeks ago, the phrase "The Restless Mind" popped into my head. So, here we go.

Personal Update

Last October, in what turned out to be the last issue of The Coffee Underachiever Newsletter, I babbled about the following topics:

- The birthdays we had just celebrated
- The garden
- My work situation
- My health
- The library database project

I'm hoping to get this out before the birthday wave, and I don't have much to report about the library database project, but here are some updates on the garden, my work situation, my health, and a few other things I want to talk about.

The Garden

We have not gotten much done with gardening over the last year, with a few exceptions: we did successfully grow a small bed of garlic. Those garlic bulbs are harvested, dried, and trimmed up, and we've got a nice supply going into the fall. Some of them are a on the small side.



Figure 1: “Our August 2024 Garlic Harvest”

It probably isn't a year's supply, but it should last us for a number of months. Grace and I celebrated by each eating a whole raw clove. This purple Vietnamese variety is *quite* pungent!

We have also grown a few things in pots, that did not require a lot of standing work or weeding. Some friends have donated tomato plants, so we've had a few of our own tomatoes, although not a lot. Our existing garden beds are, largely, overgrown disasters, with one notable exception — the raspberry bushes! Well, they are overgrown, and they need a severe trimming, but we are getting a lot of raspberries from them.



Figure 2: “Raspberries and Roses”

We recently harvested enough to flavor a batch of homemade raspberry ice

cream.

All in all, we are not close to having the kind of garden beds we had in Saginaw, and even farther from having all the garden projects going that we would like to have, but there have been some bright spots.

Work

I have fortunately had a little bit of stability in my work for a while. I started this current contracting job almost two years ago, in early December 2022. I have been fully remote since starting this job. I have never met any of my co-workers in person. I spend a chunk of every day in online meetings, and I do spend some time occasionally making presentations, but it is not a so-called “e-mail job” or “spreadsheet job” as I don’t actually spend very much time writing e-mail messages or working on spreadsheets. I do software development and debugging.

I develop virtual machines, based on the open-source QEMU project, but heavily customized to emulate specific hardware used in aircraft. Recently I have had to write code to emulate ARM instructions that are not handled by KVM mode on Linux running on ARM servers. If you don’t know what that means, that’s OK. I always feel like I should know more, too, and I’ve been doing this kind of work for decades.

Our scrum teams have been reorganized several times. Management has been reorganized several times. So I have changed projects many times, although there are commonalities that I’ve gotten to learn. The work is often tedious and frustrating, as much of what I’ve been working on is R&D — we don’t always know if we will be able to get things to work or not, and sometimes the answer is “no,” or “yes, but the performance is unusably slow.” I’ve gotten better about taking that kind of thing in stride. For the most part, my co-workers have been very supportive and easy to work with, which is not something I take lightly, as in my experience it is pretty rare.

This remains an hourly contract job. This means that we don’t have access to good health insurance, although I am paying for long-term and short-term disability insurance. So, we are attempting to “self-insure” by saving up money for health care contingencies.

This works as long as our medical expenses are far less than the thousands of dollars a month it would cost us to purchase insurance comparable to what we had through employers, which has been the case so far. But of course it leaves us exposed to great risk in the event that anyone has an expensive health issue, a scenario that becomes more likely over time as Grace and I get older, and as the social determinants of health in the United States continue their decline.

My work arrangement also means I have no paid time off for illness or vacation or anything else. I’ve been trying to establish a pattern where I save up a bit each week for 11 weeks, and then take three days off. So I took three days off at the end of March, and three days off at the end of July. I’m planning to take

three more days off at the end of September and have saved up enough to cover our usual expenses for those unapid days. Things get more difficult at the end of the year, though.

Over the week between Christmas and New Year, my employer pretty much shuts down, so even if I signed in, there wouldn't be anyone to talk to, and I couldn't get a lot done. I will probably need to take six or seven unpaid days off. That means saving up a lot more to cover our usual expenses in the weeks when I will get one short paycheck and one paycheck of zero. We will figure it out. The fact that we *have* emergency savings we can dip into when necessary is a sign that our money situation is more stable than it was a couple of years ago.

Is there a chance I could turn my current job into a permanent position rather than a contract gig through a third-party? Well, there's a chance, but it's not clear that I'd *want* to. The last time I looked into it, I found that while the benefits would be helpful, it would likely involve a huge pay cut — thirty percent or more. We can't really afford that, no matter how generous the benefit package, at least not while that take-home pay is the sole income supporting nine people.

When I was last a contractor, working for Argo AI via a contracting company, the process of going from contractor to employee was pretty frictionless — I didn't have to interview, or fill out mountains of paperwork. I'm told it would not be nearly as easy to get hired by my current employer. It's also not clear if they would allow me to remain fully remote.

So if I do wind up becoming a “permanent” employee again (as if there were really any such thing), it will probably be with a different company. In the short term, I'm planning to stay at this job, if it remains bearable and they continue to pay me, through the end of 2023. After that, I'm not sure what I will do, but it will probably be a good time to make a change, given that the only reliable way I've ever found to get a pay raise of any kind is to change jobs.

My Health

When I last wrote a newsletter, I wrote about my tinnitus, hyperacusis (extreme sensitivity to sounds), gastroparesis (a paralyzed stomach) and other digestive issues, a chronic sensation of having food or something else stuck in the back of my throat, and other symptoms including tingling and burning in my extremities, joint pain, and skin issues. All of these things have been identified as common symptoms of Long COVID, or if you prefer, “post-acute sequelae of SARS-CoV-2” (PASC).

The good news is that, overall, some of these symptoms have been gradually improving, and I continue to hope for further improvement. Don't get me wrong — I wake up every day feeling like shit, with mild to moderate joint pain and all kinds of weird aches, and a lot of fatigue. My eyes are dry and burn all day. I have to keep detailed notes paper notes during my work day, to keep track of what I'm doing, which is something I never really had to do before 2022. I get

winded easily, which is still very strange given how often and how far I used to easily walk. It's hard to stand for very long. I haven't felt that I could safely drive a car for over a year now. But it's still an improvement.

I have learned that if I rinse my throat twice-daily with hot salt water for a few days, the sensation of having something stuck in my throat goes away completely. It also seems to be tied with stress and my digestive problems, so I think it is probably a symptom of reflux, possibly aggravated by eating too close to bedtime (as our meals tend to be very late).

I mentioned that I was looking into a battery-powered device for stimulating the vagus nerve, by applying electric current to the neck just behind and below the earlobes. Well, we have one of them and have had it for almost a year. The effect that it has on some of my symptoms is *immediate*, and *dramatic*. This kind of thing is widely considered "alternative medicine" and "unproven," but I have to believe that it's because it hasn't been studied properly, not that it doesn't help people, because within a minute of applying the device, my stomach starts moving normally again and everything feels much better. It "turns on" my digestive system when it has been stuck in the "off" position.

It also helps with my anxiety, and dramatically improves problems I've had for decades with my temporomandibular joints, where it seems like I can never get my jaw to sit in a relaxed position, and it often twitches or trembles, and I wake up having been grinding my teeth. That completely goes away and I can actually relax my jaw, and it just sits there relaxed. That fixes a problem I didn't realize it could fix, which has been with me for decades. It's quite remarkable.

However, I can't recommend this particular device. I'm not even going to mention what brand it is. Why? Because of the company's business model.

Business Model Bait and Switch When we first bought it, it came with two dozen pairs of electrodes — small clear nubbins that look like gumdrops without sugar coating, and which are packaged in little sealed jars, moistened with an electrolyte solution.

The idea is that you put in a fresh pair and use the device. Over the course of weeks or months, depending on the humidity, the electrodes dry out and shrink and gradually stop working. You can slow this process down by applying a bit of extra electrolyte solution — the package came with a tiny bottle with an eyedropper for this purpose. But eventually, they don't pass much current any more, and you have to throw out the visibly shriveled-up gumdrop electrodes out and replace them with a new pair.

It seemed like an odd design, as most medical devices like it don't use gumdrops like this. I thought that maybe the idea was to make it more convenient and more portable, and to make it so that using it required less preparation and mess than a device that works with electrolyte gel in a tube. But it also didn't seem like a big deal when we bought the device, since it came with two dozen

pairs of the the gumdrop electrodes, and you could buy more by the dozen (or at least a half-dozen; I can't remember for sure), and they weren't very expensive. They have lasted a long time, as well — we still have almost a dozen pairs left. So in a year we've only used half the supply it came with.

But lately, I was looking into what it would take to buy more. It turns out that the company now has done a bait-and-switch and is operating fully on a razors-and-blades model, maximizing the cost of the blades. You can only buy the electrodes by the pair, and they want \$15 a pair for them now. (At \$15 a pair, the two dozen pairs the device initially came with would cost almost as much as the device.)

They'll offer a slight discount if you subscribe, and get a little plastic jar with two electrodes in it delivered once a month.

I'm not sure what happened, but I strongly suspect that there was a management shake-up, or a buyout, or *something*, and a manager looked over the product line, and said "What's this? Reasonably-priced supplies? Gillette didn't become a multi-billion-dollar company by selling reasonably-priced supplies! Let's quintuple the price of electrodes, and sell them only in packs of two!"

This represents a ridiculous price increase in the cost of supplies, and an incredibly wasteful and environmentally costly way of delivering those supplies. So, we'll choose to be less wasteful and throw the device in the trash when our current stash of supplies runs out, and look for an alternative product from a company that does not seek to attach itself like a leech to our bank account and suck out money each month.

Earplugs For my extreme sound sensitivity, I've been experimenting with a couple different kinds of earmuffs and earplugs.

For maximum quiet, I will sometimes put on 3M Peltor X4 earmuffs. They help a great deal, but they aren't comfortable to wear for long periods of time, and I can't really engage in conversation at all if I'm wearing them.

Loop earplugs are very small silicone earplugs that allow some sounds through, but attenuate the sound level enough to be helpful. They are comfortable and fit in a small pocket-sized case. I have been wearing them when I have to be around the kids, or I'm at my desk and the kids are making a distracting amount of noise downstairs. They don't give me silence, but they lower the sound level to the point where I find it less distracting. I can watch movies with them in, especially since I always turn subtitles on anyway, because at home, there is always noise. I would recommend these if you want some discrete, comfortable earplugs that will take down the sound level somewhat.

Recently, Grace got me another option to try: a Flare Audio product called Calmer. They aren't exactly earplugs in the usual sense — they are hollow, open tubes. They don't actually lower the volume level that I hear. They supposedly

reflect sound more accurately into your ears. This bypasses the irregular bends of your inner ear that would otherwise cause chaotic sound reflections and therefore resonate in your ear canal.

However, not only does the company not cite any studies or measurements to back up this claim, but they are using the term “inner ear” incorrectly; the “inner ear” lies deep in the ear, well *behind* the eardrum, while this device fits in the ear canal, part of the *outer* ear.

So, I am very skeptical of this explanation — after all, the human auditory processing system evolved in tandem with the human ear canal, and if there are “chaotic sound reflections” that cause a “trigger response,” it seems to me that they more likely come from our living environments, not from our ear canals.

Despite this, I am still willing to believe that the devices may be helpful, just not for the reasons given. In my home studios, I have put up sound-absorbing panels that cut down on reflected sounds, which improves the clarity of the sound recorded by my microphones. If this device cuts down on sounds of certain frequencies, it may in fact produce a subjective improvement in my experience in noisy environments.

So far, I can tell that these things *are* altering what I’m hearing slightly. It may be that they are simply attenuating high-frequency sound enough to be helpful, and it has nothing to do with my “irregular bends.” So, I can’t really recommend them, but I am continuing to experiment with wearing them to see if I get any benefit.

Better Brain? I think I have previously mentioned that since my long COVID symptoms got bad in early 2022, I’ve had various cognitive difficulties, initially quite severe, but gradually better. Along with being unable to think of words, and troubles with short-term memory, concentration, and executive function, I’ve been unable to read. As an avid reader, this has been an especially troubling symptom for me.

I don’t mean that I literally can’t look at a page and figure out what the words mean — I mean that I can’t get through any significant amount of reading before the words cease to mean much of anything and I realize I have to give up.

In previous years I’ve described taking pride in being able to complete over fifty books a year. But over the last couple of years I’ve been able to read very little. I’ve finished only a very small handful of books, and they were short and not very challenging.

Recently, though, I have found that I am a bit better able to read. One night I started reading *Goliath: Life and Loathing in Greater Israel* by Max Blumenthal — a dense book, not light fiction — and I was surprised to find that I was able to read and understand the first 40 pages in one session. I’ve been slowly chipping away at *Operation Luna*, a follow-on to *Operation Chaos*, two or three pages at a time. But recently I’ve been able to read chapters in one sitting.

So what's changed? I can't say for sure. Is this just a case where gradual healing reached a threshold I could notice?

I did recently start to take 120mg of grape seed extract daily along with my other over-the-counter enzymes and supplements. It may be helping. I can't say definitively that it is, though, as I don't have a control arm in this study I'm running on myself. Will it last? I'm not sure. In any case, I'm grateful to be closer to the way I used to be.

The Fall Crash Comes Early Despite feeling better about some things, my mood has been crashing; this happens consistently as the days start to get shorter. The magnesium I'm taking helps, and the vagus nerve stimulation device helps, but mood-wise, and stress level-wise, I'm not doing well this week.

I'm trying to go off my blood pressure meds for a while, monitoring my blood pressure daily, and see if I notice any difference. I'm not too concerned about my blood pressure. Back when I started this medication years ago, it was only elevated by a few points. It was well-controlled with medication, but I haven't really been testing it much since my long COVID symptoms started, and long COVID is often associated with dysautonomia conditions that *lower* blood pressure. So I've been wondering if it is now too low, and this might be the cause of some of my symptoms.

Testing my blood pressure daily since going off medication, it's been well within the safe range — in fact, a bit lower than usual for me. So I'll just continue to check my blood pressure myself and keep track of how I'm feeling.

Update: I just learned yesterday, September 18th, that all of the contractors on my team are being “released” — fired. I don't know exactly when, but presumably soon. So, I'm meeting with team members doing “knowledge transfer,” trying to brief them on everything I am working on. They aren't pleased with this as fully half the team members are contractors, and all of us have plenty to do. I'm updating my résumé and wondering just when I might learn exactly when I'll be finished. At the end of today? In a week? I have no idea.

Update 2: a few days later, September 24th, some of the contractors on our team are gone, but two of us are left. Our contracting companies still haven't been informed when *our* contracts are ending. Meanwhile, my employee co-workers are being told they must start taking mandatory furlough (unpaid time off). Trying to plan how to work together under these circumstances is difficult to say the least, as no one knows what next week or next month will look like. I'm going ahead and taking my pre-planned 3 days off, and maybe by next week I'll know when my contract will end. Given the circumstances it is going to be difficult to enjoy my days off, as I'll need to update my résumé and start looking for my next job, but I will do what I can. My current contracting company may be able to line something up for me, as they have an incentive to keep earning money from my work.

Family Update

Here are a few notes on raising the seven Potts children living in our household.

Elanor and Michigan Medicine

Because SARS-CoV-2 is still circulating, and mutating, and in fact transmissions and infections have been high for months, our summer has not gone quite as planned. We were hoping that there would be a drop in infections which would allow us to feel safe enough to do several previously risky things, like take Elanor to a cardiology appointment. We had already rescheduled her visit once, resulting in a delay of several months, and didn't feel that it would be right to do so again. Also, we really did feel that it was important to get early warning of any possible problems with her heart — before she showed any symptoms.

Knowing that the University of Michigan hospital system was refusing to take any and all infectious disease precautions — and even abandoning precautions they would have been readily willing to take, pre-2020, for cancer patients or other immunocompromised individuals — Grace took precautions for herself and for Elanor. We ordered a child-size, battery-powered PAPR — a “powered air purifying respirator” — and had Elanor wear it to her cardiology exam.

We've put too much love and effort and sleepless nights and money into maintaining Elanor's health, going way back to the extra care we had to give her as a premature baby, through her successful open-heart surgery as an infant, and through months of follow-up and sleepless nights administering medications around the clock, to want to risk that health now. So Grace took Elanor to her appointment in a bright yellow hooded PAPR, carrying a separate battery-powered air cleaner. The only thing we could have added, but didn't, was some portable near-UV devices, as we didn't have any, and also as this was already a lot of gear for one person to manage.

We achieved our goal — Elanor got her cardiac ultrasounds. Her heart looks quite healthy. Of course, we knew it had to be pretty healthy already, as she is a very active little girl, and extremely strong, as she demonstrates when she doesn't want to have her face washed. But it is good to have verification. And no one showed any evidence of an acquired respiratory infection afterwards, SARS-CoV-2 or otherwise.

Of course, nothing can ever go entirely well. In advance of this appointment, because we have no health insurance, we got a written estimate from Michigan Medicine. I set aside money to cover it, over the course of several months. But when we got the bill, it was for more than twice the estimate.

It took months before we could get that resolved. I was ready to literally ask the Federal government to intervene — you can do so at this web site. But it didn't come to that — because the billing codes they used to write the estimate were completely wrong, and it was their fault, they agreed to take a write-off and accept the amount of the original estimate.

So we've paid that. But if she needs a similar exam in the future, it will cost a lot more.

A Solution for Haircuts

One of our frustrations of the pandemic era has been the inability to get professional haircuts. I myself haven't had my hair cut by a professional since perhaps 2017 or 2018 — honestly, I can't even remember. I was growing it out, it was longer than I've ever worn it before, and Grace was willing to trim a few inches off once in a while, but recently I haven't been really happy with it — the split ends, the frizz, the broken-off hairs, the hair that blows in my mouth and eyes at the slightest provocation — and even when I tie it up with a clip or elastic bands, there are always shorter stray hairs that fly around.

Grace has been able to go to a salon once or twice since the start of the pandemic, but the kids hadn't gotten haircuts other than an occasional trim with clippers at home.

Recently, we tried a rather spectacular solution to the problem. Grace actually hired a couple who operate two portable salons, in retrofitted airport shuttle busses. So one fine summer day they both parked their salons/busses in the driveway and we took turns getting haircuts — with air cleaners running, UV light kits running, and everyone masked.

The haircuts were great. Here's Sam:

Grace asked me if I would get my hair cut to the length that King Théoden wore his hair in *The Lord of the Rings*. King Theoden was played by Bernard Hill, who sadly died a few months ago. So that's how we got it cut. (And, yes, I know, the actors were wearing expertly-styled wigs, not their own hair, but lacking a wig department, we do what we can).

This haircut does in fact make me look, just a bit, like King Théoden. I'm hoping that I won't die crushed under a fallen warhorse. RIP, Snowmane:

Faithful servant yet master's bane,
Lightfoot's foal, swift Snowmane.

Everyone looked great, and I think it helped all of us feel better, but working with the traveling hairdressers was quite expensive. I'm glad we did it, but I'm not sure we're going to do it again anytime soon. I guess if you amortize the amount we spent over several years and many people, it really didn't cost that much. Maybe we can save up some money and do it again next year.

Oh, I almost forgot. Malachi and Elanor did not get haircuts, but they would like to say hello:



Figure 3: "Paul's Hair"



Figure 4: “Mobile Hair Salons”

In Case You Hadn’t Noticed, Everything is More Expensive

Although the Very Smart Pundits and Economists are all busily lecturing us, telling us that inflation is slowing and so we have nothing to complain about now, those of us who actually work to earn money continue to notice that everything costs more than it used to — in some cases, a lot more. They think they can bluster and bullshit you into failing to notice that “slowing inflation” just means that the costs of their benchmark goods and services are rising a bit more slowly than they were. But my *income* hasn’t changed in some time, and I think most Americans are in this boat.

There’s also the minor matter that a great deal of the increase in the costs of consumer goods have nothing to do with inflation, but instead come from companies using inflation as an excuse to raise prices (and profits) to record levels. If it was the case that food suppliers were simply passing on their own inflated costs to consumers, their profits would remain relatively flat. But this hasn’t been the case:

As food costs have skyrocketed for Americans, some of the country’s biggest chains and grocery brands, including General Mills, PepsiCo, and Tyson, have blamed the price hikes on supply chain issues and economy-wide inflation. But behind the scenes, these companies have expanded profits and quietly authorized billions of dollars in lucrative stock buyback programs and dividend payouts to shareholders.

A few months ago I was startled to find that our monthly consolidated mortgage payments were going to increase — by a lot. There was apparently a large deficiency in the escrow account. It turns out that this deficiency is *entirely* because our homeowner’s insurance increased, without any other notice or



Figure 5: "Sam's Haircut"



Figure 6: “Paul’s Haircut”



Figure 7: “Malachi and Elanor”

warning, by *eighty percent*. Grace got on the phone with Liberty Mutual to discuss this. They mumbled something about how their costs have gone up. They offered us a \$100 courtesy discount (which, on a premium of \$5086.00, amounts to less than two percent).

Our response? Fuck you very much, we’re looking for another insurance company.

Feeding Teenagers

With four steadily growing teenagers, we have had to come up with strategies for controlling costs. Some have worked well, and some haven’t. Below, I describe the strategies we are currently relying on to feed our family. Managing our household is a bit like managing the inventory of a small restaurant. We’re constantly trying to find the best suppliers, balancing quality against price.

I should probably mention that, perhaps once a month, when nothing is going right, as far as getting a meal together, and everyone is tired and cranky, and the kitchen is not in usable condition, we wind up breaking down and ordering pizza and wings for the whole crew, to our shame and chagrin. Or, if we’re feeling up to driving to pick up take-out food, we get three big trays of grilled chicken and vegetables from Palm Palace — altogether a healthier choice, and actually cheaper than the pizza meal. But we try to let these lapses happen as rarely as possible.

Bulk Items from Webstaurant In addition to the shelves with canned goods and packaged food, we’ve got eight large restaurant-style food storage containers in our basement, each with a big stainless-steel scoop in it, as well as six smaller

containers for shuttling things up and down the stairs. Our utility room looks a bit like a bulk food store:



Figure 8: “Bulk Food Storage”

Top row, left to right: granola, brown rice, chopped peanuts, rotini and orecchiette pasta. Bottom row, left to right: steel cut oats, TVP, rotini pasta, orecchiette pasta.

We pay a \$99 fee each month to get free shipping on many bulk items from Webstaurant. I have a small “set-aside” fund — really just a line in a spreadsheet — where I set aside money each week to cover this monthly fee.

Recent orders have included:

- 20 pounds of Barilla rotini pasta
- 20 pounds of Barilla orecchiette pasta
- 25 pounds of Bob’s Red Mill granola
- 25 pounds of Bob’s Red Mill steel cut oats
- 25 pounds of Bob’s Red Mill textured vegetable protein
- 10 pounds of roasted, unsalted chopped peanuts (these are sold as a sundae topping; we throw them in the food processor to make peanut butter that is better than anything we’ve ever been able to buy)
- Half-gallon bottles of tamari soy sauce
- Assorted bulk herbal teas including hibiscus, tulsi, rooibos chai, etc.

- Assorted kinds of popcorn salt including garlic, salt and vinegar, and cayenne
- Assorted kitchen equipment including stainless steel bowls, strainers, carbon steel frying pans, and stainless steel scoops for the bulk food bins

All of this stuff came with free shipping, with our membership. Given what we would have paid for shipping, the monthly fee has been a net gain so far, although we reconsider periodically whether we're getting enough value out of it to continue.

Lundberg Brown Rice We periodically (quarterly, maybe, or a little more often now) order 25 pound bags of Lundberg short-grain brown rice, which comes with free shipping. I have a set-aside fund just for this brown rice. It's really good.

Olive Oil We have another set-aside, to save up money and periodically order olive oil in three-liter tins. The tins are expensive but it is quite a bit cheaper per ounce to buy in bulk, and the quality is excellent.

Chocolate I think I've mentioned before that we buy chocolate and cocoa powder in bulk during the fall and winter when shipping from Guittard is much cheaper (during the warmer months, they ship chocolate in heavily insulated boxes with ice packs). We're still using our stock purchased in February. There's a lot of chocolate left, as we didn't do as much baking as we planned over the past year, but our cocoa powder is almost gone, as the kids frequently make hot cocoa before their bedtime, even in summer.

Tea We get Harney and Sons tea in bulk. Recently, my weekday breakfast has been a matcha latte, made with oat milk. I have been getting a can delivered every six weeks. The matcha is a little less stimulating than coffee (although don't get me wrong, I will still have a small coffee occasionally), and seems like it has some health benefits.

We also occasionally get bulk bags of other teas such as Malachi McCormick blend, Grace's favorite. And we've experimented with their ground chaga mushroom powder. It is relatively cheap and *definitely* has very noticeable medicinal effects, giving me immediate improvement in joint pain and other long COVID issues, but I've been hesitant to order more because it tastes pretty nasty and is hard on my stomach. It also seemed to irritate my urinary tract quite a bit, which is an unusual sensation I wasn't expecting. There are apparently some reports of oxalate nephropathy induced by "excessive" chaga intake — in the case of the study mentioned, ten to fifteen *grams a day* of the powder. My intake, eight ounces of tea a day made from one teabag of chaga powder, taken for only a week, hardly seems like it ought to be enough to be toxic, but I found the symptoms concerning enough to stop using it. Could it have a bad interaction

with the 1-gram capsule of vitamin C I was taking per day? I don't know. For now I have stopped using both the chaga and the vitamin C.

Dr. Bronner's Products The local co-op has not been able to consistently order large packages of Dr. Bronner's Sal Suds. We buy Sal Suds by the gallon and use it for laundry and dishwashing. We've also been getting bar soap (for hand and body washing) and sugar soap (for face-washing) directly from Dr. Bronner's. I have another set-aside fund for periodic orders.

Recently, I tried washing my hair with the sugar soap and rinsing with their hair rinse. The rinse is slightly acidic, made with lemon juice; you heavily dilute it in water before use, then rinse it out.

The combination leaves my hair very nice, smooth and glossy, but unfortunately I had to stop using it, as my scalp was becoming raw and sore. The same thing happened when I used apple cider vinegar, again, heavily diluted, then rinsed it out. I've been forced to conclude that for some reason my scalp just can't tolerate even brief exposure to anything acidic.

Local Meats Vestergaard Farms will deliver frozen local meats. The quality is very high, although the quantities are a bit problematic. We've been getting "sampler" packs which are cheaper by the pound (I think they package up whatever items haven't been selling well at full price), and come with various types of meat items, but typically not enough of any one kind of meat to make a meal for nine out of it. We have been working around this by ordering multiple sampler packs and accumulate meat items in our freezer until we have enough of, say, pork chops, to make a meal for nine. We have also been stretching ground meats with TVP (we're in good company; Taco Bell has been doing it for decades).

Vestergaard Farms also stocks a locally roasted whole-bean coffee we like, so we get that occasionally as a treat. I grind it with my hand grinder and make a press pot to drink on weekends.

Local Vegetables Currently, during the season, we are getting bulk produce deliveries from Argus Farms. Again, the quality is very high, but the quantities are a bit problematic. Compared to grocery-store produce the price is excellent. To fix the quantity problem we're paying for two orders a week. Probably, almost all the households who sign up get a single order, but that just isn't enough food for our household of nine.

Here's a recent double order that included a couple of gorgeous fennel bulbs:

Fresh fennel bulbs are something I love but have not had success growing myself — I always get to them too late, when they are not tender — so this was an especially welcome treat. I made a big cast iron pan full of roasted fennel, potatoes, and squash, drizzled in olive oil, and topped with salt, pepper, and fresh thyme:



Figure 9: “Argus Farms Vegetable Double Order”

It was fantastic — what a wonderful aroma! To my surprise, the kids cleaned out the pan in minutes.

We also get produce from the local Farmer’s Market when we can, and the hyper-local farm stand just down the street, Nemeth Orchard. And, last but definitely not least, we’ve also been blessed by friends who have been sharing their bumper crops of things like homegrown tomatoes, and even treats like shishito peppers. Yesterday — Sunday, September 15th — we made a huge pot of spicy homemade tomato jam, so as to use up a whole lot of tomatoes that were very ripe. It turned out really well!

So we’ve been getting the full late-summer food experience, and we’re eating some of the best produce available anywhere, which I look forward to every year!

Calder Dairy Due to H5N1 we have restricted the dairy products that we buy. We have not been buying any fresh milk, butter, sour cream, or half-and-half for some time now.

We are still buying milk in several alternate forms, although not from Calder Dairy: sweetened condensed milk, ultra-pasteurized heavy cream, and ultra-pasteurized half-and-half. Interestingly, Costco is stocking several new ultra-pasteurized milk products, so apparently we’re not the only one with a concern about the safety of milk products. We’re getting imported butter occasionally, but for baking only. From Costco, we have been slowly — slowly, because they are expensive — stocking up on aged cheeses produced before the current outbreak began, particularly Pecorino Romano and Parmesan Reggiano, both of which freeze well. We will soon be out of “runway” and will have to suspend purchase



Figure 10: "Roasted Fennel Dish"

of all cheeses, but we will have some aged cheeses we can use occasionally in pesto or pasta.

Unfortunately it does not appear that the country is coming anywhere *close* to taking the necessary steps to solve the bird flu situation.

All this means we aren't using Calder Dairy as a dairy any more. But we want to support them, and they do have other products we want. We continue to get a weekly delivery of eggs — we go through about five or six dozen eggs a week, depending on the week — and a few special once-a-month items like pickled beets, pickled green beans, honey, jam, maple syrup, and some very nice locally produced popcorn.

Costco Although we'd like to phase out Costco, because of their business practices, we still find them indispensable, so we still place a Costco order each week, for delivery via Instacart. Typically we get some of the following items:

- Oat milk — typically, two or three cases of six cartons
- Bread — typically two loaves of sourdough and four loaves of whatever reasonably good whole grain loaves they have in stock
- Other carbs — corn chips, flour tortillas, bagels, Brownberry Sandwich Thins, etc.
- Occasionally: a treat for Sunday morning like cinammon rolls, strudel, croissants to stuff with chocolate, etc.
- Dehydrated hash browns — Pippin makes hash browns and eggs for breakfast almost every day
- Fizzy water — Waterloo or LaCroix as a treat for our Saturday family movie nights
- Pre-cut vegetable trays (sometimes) for movie night meals
- (For now) aged cheese to stock our freezer
- Ultra-pasteurized heavy whipping cream and/or half-and-half
- Hummus
- Toom brand garlic sauces when it is available
- Japanese barbecue sauce when it is available
- Packaged salad kits (sometimes)
- Avocado mayonnaise
- Ghee
- Avocado mash
- Safe Catch brand canned tuna
- Fish sticks (depending on price and availability)
- Salmon burgers (depending on price and availability)
- Bulk frozen french fries (depending on price and availability)
- Frozen fruits (depending on price and availability)
- Canned coconut milk
- Diapers, toilet paper, baby wipes, and paper towels as needed
- Water softener salt as needed

Gordon Food Service From GFS, for curbside pickup, we typically buy the following:

- Large cans of crushed tomatoes (the kids go through these fast, making pasta sauce for themselves)
- Large cans of black beans
- Large cans of refried pinto beans
- Small cans of refried black beans
- Corn tortillas (Costco doesn't have them)
- Bulk coleslaw mix
- Frank's hot sauce
- Bustelo ground coffee (this is our cheap "backup" coffee and we keep several pounds on hand in the event of total societal and financial collapse, or just a really bad week when we need to use the drip coffee maker to make big pots of coffee)

Sam's Club From Sam's Club, for curbside pickup or delivery, items as needed or when they aren't available elsewhere (not usually weekly; most weeks we don't order from Sam's Club):

- Sliced ham
- Potatoes
- Onions
- Carrots
- Packaged salad kits (sometimes)
- Spring Mix *aka* yuppie chow
- Fish sticks (depending on price and availability)
- Salmon burgers (depending on price and availability)
- Bulk frozen french fries (depending on price and availability)
- Bulk frozen chicken strips (some days, certain kids will eat these when they won't eat anything else)
- Ground turkey

Target From Target, for curbside pickup, we sometimes get:

- Bustelo instant espresso powder (for frequent use making quick lattes or mochas with our frother)
- Canned chick peas (they have the best price) — we eat a lot of chick peas
- Coconut sugar
- Alcohol wipes
- Certain medications and vitamins that they stock at a good price
- Vanicream shampoo
- Baking items (sometimes)
- Craft items (sometimes)
- Blu-ray movies (when on sale, shipped)
- Drain cleaner (we can't get it from Costco or Sam's Club)
- Lil' Debbie brand oatmeal "creme" pies (Veronica's personal favorite)

There's been a rumor circulating that the oatmeal "creme" pies are being discontinued. From what I could find out, this rumor is unfortunately not true.

Library Resources

We've made some improvements to the library facility over the past year. We now have two wooden tables with five straight-backed chairs, in addition to one high café table with two tall barstools, which is where I like to sit. So more people can sit and work downstairs in the library at either a low table or a high table as they prefer.

I also bought some calculators for the library, described below.



Figure 11: "Library Calculators"

The TI-30X Pro MathPrint We now have three TI-30X Pro MathPrint calculators in the library, and I also got one for my personal use, to carry in my "math bag." Why personal use? That will become clear below when I describe what we are doing with Math Academy.

These aren't graphing calculators, but they allow you to enter algebraic expressions very easily, and they will handle a lot of expressions in exact form without converting them to decimal.

Although I have owned a few over the years, I don't actually *like* any of the recent generations of Texas Instruments graphing calculators, as they have extremely ugly and inelegant user interfaces, compared to their non-graphing counterparts. I did get Sam a Texas Instruments CXII TI-Nspire CAS Graphing Calculator, but I don't use one myself. I use Wolfram Alpha (the free version) sometimes, or the wonderful Desmos graphing calculator.

Why don't I like the graphing calculators? They run applications, in an ugly way, like Windows CE, without the elegance of the Apple Newton or Palm Pilot, and the hardware is overpriced and dated (for example, none of them support USB C).

If I want to play with a hardware graphing calculator, I have an old Voyage 200 that I bought on eBay not long ago. Despite being dated and slow (it's from 2002), it's pretty cool — it certainly has a more elegant user interface than the recent graphing calculators — but it lacks the MathPrint features that I like a great deal, and is still app-based.

Are there better graphing calculators from other companies? I'm not sure. But for now we're sticking with these non-graphing models.

The TI-30X Pro MathPrint will handle imaginary numbers and complex numbers. For example, if I tell it to evaluate the square root of negative eighteen, it won't show me a number, but will instead show me an exact answer: three times the square root of 2 times i . If I want it in an approximate number form, I can press one key and it will display 4.242640687 times i . These calculators will also handle derivatives, integrals, and other higher math operations. As far as I'm concerned, these are the best TI calculators I've ever used. The design is simple, light, and elegant, just as complex as it needs to be, and no more complex. I especially like the "MathPrint" templates for easily entering mathematical expressions. This isn't a brand-new feature, but this model seems to have added more templates and made them easier to recall and edit.

Unfortunately, this model is not sold in the United States, so I had to order them from an eBay seller who imports them from the UK. If you search for "TI-30X Pro MathPrint", make sure you get an exact match, because there are similar models with lower-resolution screens, reduced functionality, and slower processors. Don't get the 30X "Plus" model, or the "MultiView" model, but "TI-30X Pro MathPrint." Accept no substitutes!

The TI-34 MultiView We also keep three TI-34 MultiView calculators in the library. This is an older model of TI calculators with a target audience a few grade levels behind the target audience of the TI-30X Pro MathPrint. This model does not handle derivatives and integrals, or complex numbers. I wanted an "intermediate" calculator for the younger kids and that's just what this is. It has some support for "MathPrint" templates.

To get four of these, three for the library and one for me, so I can learn how

to demonstrate them to the kids, the cheapest option was actually to buy a teacher’s set of ten on eBay, which came in one big box, and to order a set of sliding covers. So now we have ten.

If you homeschool kids and you want some yellow TI-34 MultiView calculators, the yellow model that says “SCHOOL PROPERTY” on it, get in touch!

Recent Acquisitions — Books

I should probably keep a separate spreadsheet listing everything that I acquire for the library. Maybe I’ll start one now. But meanwhile, here is a list of some of our recent acquisitions — specifically, some, but not all, books we’ve received from Alibris and eBay sellers over the last year, intended for the library, as opposed to books acquired for individuals:

***General Relativity: The Theoretical Minimum* by Leonard Susskind**

I have two complete sets of these *Theoretical Minimum* books, a hardcover set and a set of the imported Penguin paperback editions. In these books, Susskind and various coauthors explains the mathematics necessary to understand modern physics, beginning with the first volume, which covers classical mechanics. This is the most recent volume. I cannot yet understand this book very well, but I hope to one day, after working my way through the previous volumes.

***Swords and Deviltry* by Fritz Leiber**

I’ve long been aware of the Fafhrd and the Grey Mouser stories by Fritz Leiber, but the fact that I never read them has left a real gap in my knowledge of science fiction and fantasy. I’m trying to remedy that now. I wound up getting this copy for free, as it arrived badly damaged and I got a refund. This sometimes happens when sellers package books in padded envelopes instead of boxes. It is still readable, but the spine was broken and some pages were slightly torn, so it’s now what you’d call a “reading copy.” I have a replacement on order from a different seller.

I have always advocated for shipping books in boxes instead of envelopes, padded or otherwise, because books shipped in envelopes very often arrive damaged, but books shipped in boxes very rarely arrive damaged. The USPS recently managed to “prove the rule” (which doesn’t mean what people think it means), by delivering a Library of America book in a box that was *also* badly destroyed. We got a free replacement copy of that book, as well.

***Goliath: Life and Loathing in Greater Israel* by Max Blumenthal**

Blumenthal is a frustrating figure for me — he’s incredibly well-informed on foreign affairs, including the conflicts in Israel and Ukraine. So I consider him a go-to source like Noam Chomsky, Chris Hedges, and Norman Finkelstein. But on domestic questions he often takes uninformed, reactionary, libertarian positions. So I ignore anything he says about COVID, and Trump, and “culture war” issues, and instead read (and listen to) his analysis of the Gaza genocide with great

interest. In this book, he recounts years spent living in Israel and what it taught him about both the history, and especially, the *culture*, of modern Israel.

***Real World OCaml: Functional Programming for the Masses* by Anil Madhavapeddy and Yaron Minsky** I'm investigating OCaml to see if it makes a viable alternative to Haskell, which has gotten monstrously complicated over the years, and now seems to be used primarily by programmers who specialize in obfuscation and condescension. This isn't to say Haskell is bad, but that the culture around it is not to my taste; modern C++ has similar problems, in that it seems to be designed by, and oriented towards, Ph.D. candidates than working software engineers.

***String and List Processing in SNOBOL 4: Techniques and Applications* by Ralph E. Griswold** The SNOBOL family of programming languages are long-gone, and yes, the name is a play on COBOL, but they are far from forgotten. Features of the language live on in many other languages that use regular expressions and pattern matching, including Python, Haskell, and many others. I've been looking for an affordable copy of this used book for a long time and finally found one, when an automatic eBay search found me a copy. It's in excellent shape except for the fact that it has no dust jacket.

***The Inform Designer's Manual* by Graham Nelson** The Inform programming language is a modern version of the languages originally created to make *interactive fiction* games — that is, text adventure games. I have had a fascination with these games going back to the late 1970s, and as a kid, tried to write my own, creating a few small working experiments, now long-gone. I'm hoping to get at least one of the Potts kids interested in writing interactive fiction.

***Beren and Lúthien* by J.R.R. Tolkien (Unabridged Audiobook on CD)** I thought “if I can't read, can I listen?” The answer is yes. I have read much of this material in other forms, as some of it was in the multi-volume *History of Middle Earth* books, but it's nice to have it assembled into the form of a continuous narrative. Much of what Tolkien wrote was never truly polished into a final form, but Christopher Tolkien did a respectful job here with the incredibly complex and messy jumble of drafts that Tolkien left behind. It's still uneven, as portions were written in different styles across decades and Tolkien himself never turned them into a coherent single story, but Tolkien's imaginative, if inconsistent, world-building more than makes up for the messiness.

***That's Blaxploitation!: Roots of the Baadasssss 'Tude (Rated X by an All-Whyte Jury)* by Darius James** This book by the author of *Negrophobia* is a critical, opinionated personal labor of love examining the Blaxploitation genre of film. I have a love-hate relationship with these films, such as *Shaft* — they elevate black characters and actors, but playing with stereotypes is often a

risky venture. Sometimes they work really well to overturn them, and sometimes they wind up affirming them. Some of these films are simply great by any measure, and some aren't. In any case, any cinephile should be familiar with the genre.

***Programming in Lua, Fourth Edition* by Roberto Ierusalimschy** Lua is a lovely, quirky, simple, highly-efficient language that has found a niche as an embedded language used in many computer games. This is for Pippin, who is working on game programming, and goes along with *The Lua 5.3 Reference Manual*, also acquired recently.

***The Trial* by Franz Kafka (Modern Library Definitive Hardcover Edition)** I really should have read this years ago, but I was far more familiar with Kafka's stories than his novels (and there are only three, of which *The Trial* is by far the most famous). I'm reading it now, out loud to Grace, a chapter at a time. We also watched the Orson Welles film version of *The Trial* recently — it's a bit of a mess, with a botched ending, but a gorgeous mess.

***The Christmas Books* by Charles Dickens (Folio Society)** I wanted a nice edition of *A Christmas Carol* to read out loud at Christmas time. This is a used copy in near-fine condition, with illustrations. It also contains several other Christmas stories by Dickens, not as famous as *A Christmas Carol*, that I was not even slightly aware of.

The Grinnell Beowulf This is a translation of *Beowulf* (I have several) which has the distinction of being written by a group of undergraduate students working with Associate Professor Timothy D. Arner at Grinnell College in Iowa. There was more to read about this edition online, but unfortunately the links I found are all broken at the moment. If you get me drunk I will attempt to read Chaucer aloud in Middle English. Old English is considerably harder, so I generally don't try.

***Gaza: An Inquest Into Its Martyrdom* by Norman Finkelstein** This is probably *the* most indispensable single book on recent the history of the Gaza Strip. I have read sections of it, but the unrelenting grim nature of the history means that I can only take it in small doses. Still, I think we are obligated to bear witness.

***The King of Elfland's Daughter* by Lord Dunsany** I've been looking for a decent copy that wasn't an expensive collectible. I've got one now and will, I hope, get to read this soon!

Fifty-One Volumes of Shakespeare, Sort Of In a series of purchases from eBay sellers over the past year or so I acquired, for the library, 48 volumes

of Shakespeare's plays — they are Pelican paperbacks of the same uniform edition, some better-printed than others, along 3 books of poems. I got these by buying several cheap lots from eBay sellers and discarding copies that were badly damaged. I have multiple copies of several of the more famous plays — two copies of *Hamlet*, six of *A Midsummer Night's Dream*, four of *The Tempest*, a couple of *Merchants*, a few Richards and Henries, and so on.

The idea here was that we'd be able to get groups of kids together and do table readings. So far, that hasn't really panned out, as the kids have displayed great indifference to this project. There's also the possibility of readings over Zoom with other homeschoolers, if I can't get my own kids motivated to read Shakespeare.

As part of a vague plan to study some of the works of Shakespeare with the kids, I've also acquired a number of movie adaptations, including versions of *Hamlet* by Olivier, Branagh, and David Tennant, *Twelfth Night*, *Richard III*, a couple of versions of *Henry V*, Branagh's fantastic version of *Much Ado About Nothing* (one of my favorite films of all time), and Kurosawa's *Throne of Blood*, a terrific and spooky adaptation of *Macbeth*. I'll probably acquire two or three versions of *Romeo and Juliet* at some point as well.

The Book of the SubGenius: The Sacred Teachings of J.R. "Bob" Dobbs As I was a big fan of the teachings of J.R. "Bob" Dobbs back in the day, I remembered this fondly, and thought that Sam might find it amusing, as it fits in with his penchant for dry humor and surreal satire. He didn't really enjoy it, though. Maybe, just as Ram Dass exhorted us to Be Here Now, to appreciate this, you had to be there, then.

The Centipede Press Library of Weird Fiction: Frank Belknap Long This is a very nice hardcover edition featuring stories by an underrated twentieth-century writer. I want to read some of these in live online readings, as they include such wonderfully creepy stories as "The Hounds of Tindalos", indispensable to any scholar of weird fiction.

This doesn't summarize everything we've acquired for the library over the last year or so — it doesn't cover every book, and it doesn't list any of the movies or TV shows, or the CDs. But, I'll end there for now, as describing everything would be a much bigger task.

Reading and Watching — September 2023

Last September, I started keeping a spreadsheet in which I wrote down every single film or TV show that we watch, at least the ones I'm aware of. I'm going to go through some highlights from last September, and I'll see if I can gradually catch up. Some of these are now in our library, in the form of DVD or Blu-ray discs. Some of these we watched on Paramount Plus or the Criterion Channel.

Movies

***Guardians of the Galaxy 3* (Library)** There are some fun scenes and weird action sequences, but overall I found this Marvel film to be badly overstuffed, a huge, sluggish mess. This seems to be true of most of the recent Marvel films, unfortunately.

***Bamboozled* (Library)** I had seen this film before, but it had been a few years. It's a flawed but still fascinating examination of American racism and the people that choose to benefit from selling it. We watched this with the older kids and they certainly found it thought-provoking. Crispin Glover's performance still amazes me.

***Gaslight* (Criterion Channel)** Yes, this is *the* film, based on the play, that led to the word "gaslight" entering the lexicon as a verb. It's a beautiful *noir*-influenced film with a great story — highly recommended.

***Babylon 5: The Road Home* (Library)** This is a direct-to-video animated film set in the original *Babylon 5* universe. As a fan of the original show, I found this to be quite entertaining and well-made. Hearing many of the characters voiced by the original cast was especially endearing.

Sadly, several cast members have died, so several of my favorite roles were not voiced by the original cast. G'Kar was not played by Andreas Katsulas, Chief Michael Garibaldi was not played by Jerry Doyle, and Ambassador Delenn was not played by Mira Furlan. So, watching this, for me, can't help but be a somewhat wistful experience. It's good, and I enjoyed it, but I also can't really imagine wanting to re-watch it, so we'll probably give this away.

***Eight Diagram Pole Fighter* (Library)** This is a terrific wuxia film; you could call it one of the founding documents of the wuxia genre. It has some of the most impressive fight choreography I've ever seen. Don't expect a deep philosophical treatise! Enjoy the wild action, glorious costumes, and scenery-chewing acting.

***The War of the Worlds* (Library)** This is the original film from 1953, directed by George Pal. We watched a Criterion Blu-ray which features amazing color restoration. It's a terrific film. By modern standards, the effects are a bit cheesy and the patriotism a bit heavy-handed. But if you can suspend your modern standards and try to see it as audiences saw it sixty years earlier, you can get a sense of how startling and influential it was. And, of course, we learn that the mad scientist character Clayton Forrester, on *Mystery Science Theater 3000*, took his name from Clayton Forrester, the not-mad scientist hero of *The War of the Worlds*.

***Dirty Ho* (Library)** I saw this recommended as a fun Kung Fu movie. Unfortunately, while it has some some great action sequences, this film features some especially offensive clownish characters that I found too tasteless and stereotypical to enjoy; I felt dirty watching it. In short, it just hasn't aged well. So, it went into the giveaway pile.

***The Last Dinosaur* (Library)** I remember watching this TV movie in fascinated disbelief back in 1977, and have long wanted to see it again. It is a Rankin/Bass collaboration with Tsuburaya Productions (a Japanese production company), a gloriously weird and cheesy train wreck of timeless science fiction tropes and horribly dated sexual politics, with laughably bad dinosaur effects, but somehow, I still had a good time laughing at it. There's another movie from about the same part of my childhood that I want to watch again, called *Bermuda Depths*, and I'll probably get myself a copy of that film at some point. I can't actually recommend it to normal people, as it is a *terrible* movie. It would have made a great episode of Mystery Science Theater 3000.

***John Wick Chapter 2* (Library)** If you can tolerate both stylized and graphic violence, you might agree with me that the John Wick series are some of the absolute best action films I've ever seen — beautifully paced, relentless, with amazing locations, characters, and fight scenes. Are they realistic? Nah, they're fantasies. But who cares? Keanu Reeves as anti-hero John Wick is a badass. I would only show this to older teens (the film is rated R). I recommend watching them in order, since the story continues between the films.

TV Shows

***Star Trek: Lower Decks* Season 4, Episodes 1 to 4** *Lower Decks*, an animated *Star Trek* series that combines both homage and parody of the long history of *Star Trek*, is great, although uneven. The first and second season were substantially better than the next two, but it's been pretty much all worth watching. Season 5, allegedly the final season, will premiere this fall.

***Years of Living Dangerously* Season 1, Episodes 5 to 7** This documentary series is interesting, and sometimes brave. Watching it, I am alternately impressed that it was actually produced, and frustrated that it didn't seem to achieve much of anything, and there is no national leadership on climate change.

***Fringe* Season 1, Episodes 1 to 3** *Fringe* is a show both inspired by, and in many ways much better than, the original *X-Files*. Season 1 is especially good. I recommend it, but it is for adults and older teenagers — there are a lot of grotesque special effects and occasional scary moments. But, mostly, younger kids won't appreciate John Noble as Walter Bishop. Even if there were no other decent actors or interesting characters in *Fringe*, and there are, Walter Bishop alone would make the whole thing worth watching.

Math Anxiety and Math Therapy

OK, now — after the preliminaries, on to my main topic for this week!

Math and Me — the Early Years

Back when I was in grade school, my mother and grandparents knew that I was struggling in the public school environment — the physical and social environments were very stressful for me, and I had physical symptoms including severe headaches. So, for a few years, they arranged for me to attend a private school called the Erie Day School, which was stressful in its own ways, but did give me the opportunity to learn from some dedicated teachers who helped me learn at a speed that was closer to my potential. In particular, I had great teachers in English, chemistry, and math. By the end of sixth grade, I had blown through all the graded reading materials, up through twelfth grade. I learned to balance equations and do basic chemistry lab work. I learned some algebra, up through the basics of quadratic equations.

During this time, I recall loving math, and I was often thinking about mathematical topics. At some point — my memory is a bit hazy — I recall reading about topology in Scientific American magazine. It *may* have been this January 1950 issue that I found in a friend’s family’s home, although I bought this copy decades later:

I remember being fascinated by the three cottages problem, described in this issue:

Although I probably first read about it elsewhere. It may have been in this Life Science Library, which I think we owned back then, although this is not the original copy, and the pages are falling out:

I recall being fascinated by Möbius Strips, and making them out of paper, and cutting around them to explore their bizarre properties:

But I also recall becoming very frustrated when I tried to teach myself topology from math textbooks — a frustration that has lasted my whole life!

After sixth grade, for reasons I probably was never fully told about, my family took me out of the Erie Day School and enrolled me in Harborcreek Junior/Senior High School as a seventh grader — and although I was eventually able to take “Advanced” English, history, two semesters of chemistry, and physics, my math education suffered a major setback, and I was again given topics I had already mastered, or at least *thought* I had already mastered — maybe I did poorly on a placement test, I don’t really recall — fractions, decimals, percentages, etc.

I got through these prealgebra topics in Junior High grades 7 and 8, and proceeded into the series of classes on Algebra 1, Geometry, and Algebra 2, although I recall being very frustrated and bored with the slow, rote presentation of this material. I especially enjoyed geometry, and in geometry, I especially enjoyed constructing geometric proofs, which required less rote work and more thought.

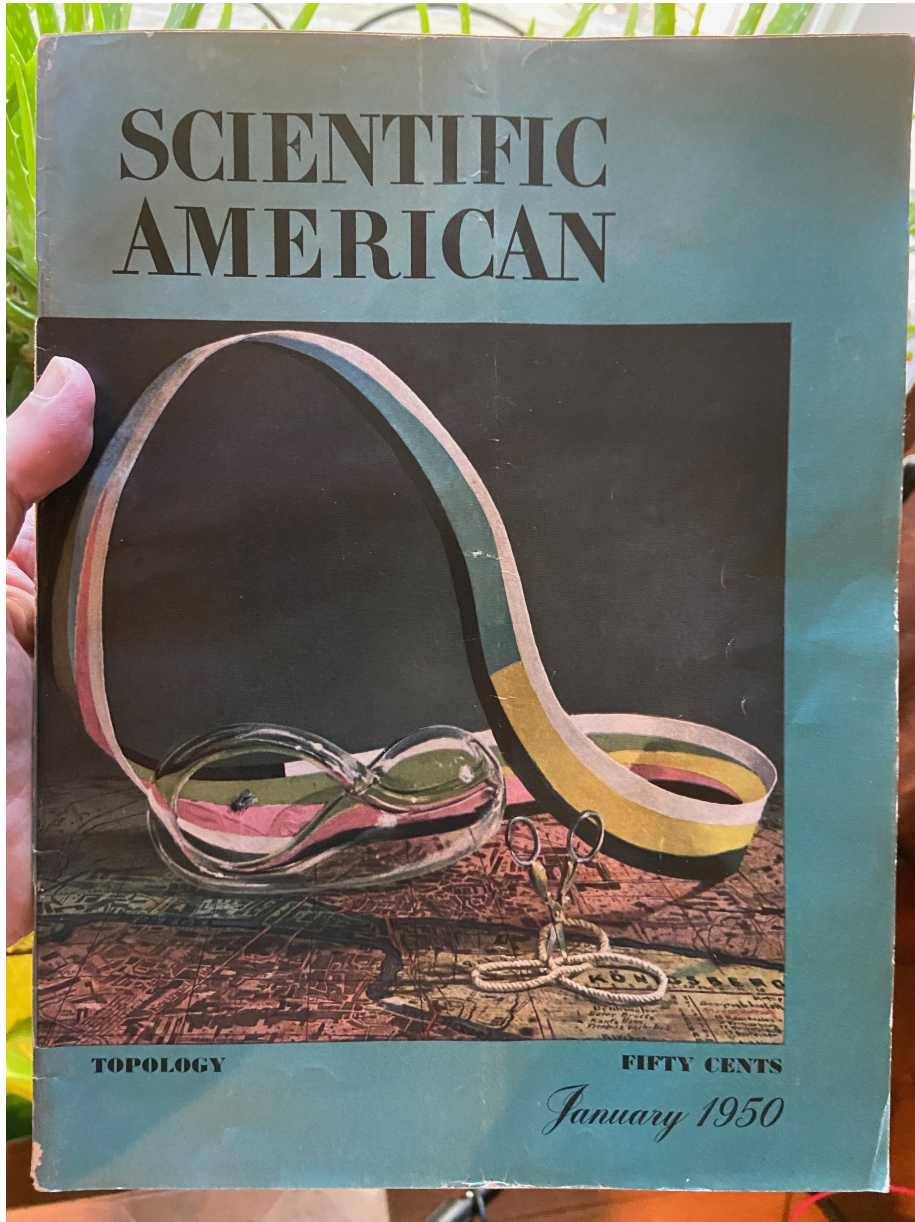


Figure 12: "Scientific American, January 1950"

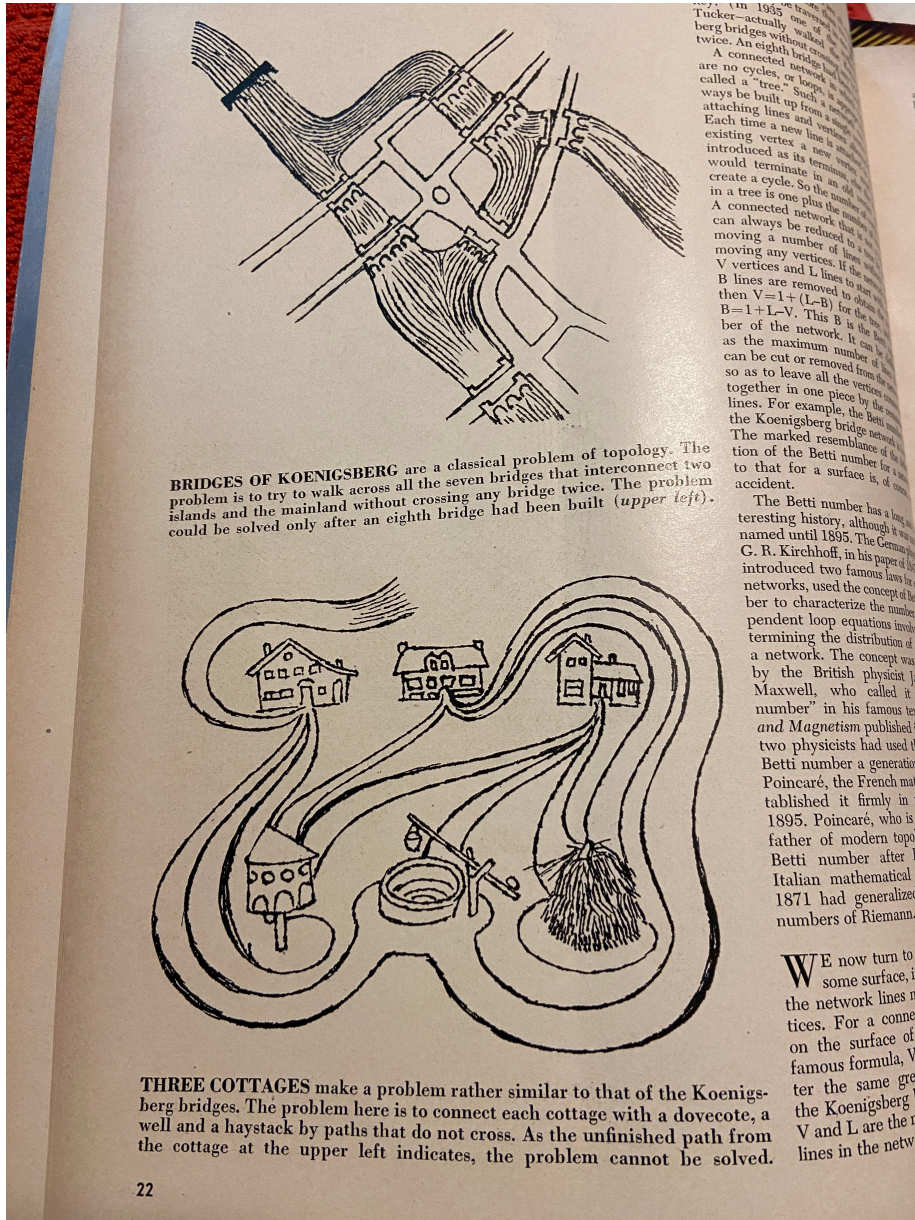


Figure 13: "The Three Cottages Problem"

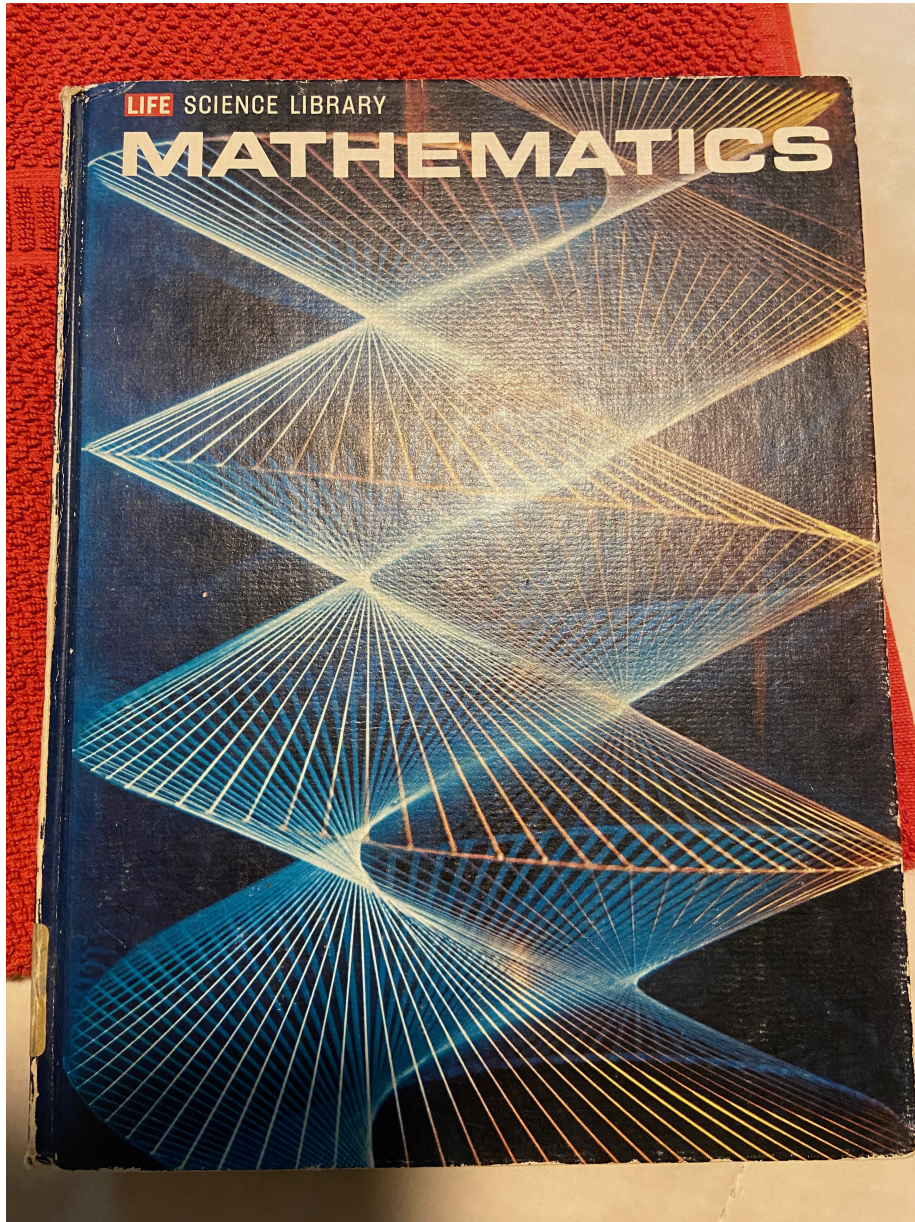


Figure 14: "Life Science Library: Mathematics"



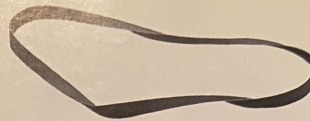
THE ONE-SIDED MOBIUS STRIP

A Möbius strip is easily made from an ordinary flat strip of paper: first the strip is given a half twist and then the two ends are connected to make a closed ring, as in the picture above.



"HALVING" A MOBIUS STRIP

When a cut is made around the middle of a Möbius strip, it might be expected to divide the strip in two. But when a line is drawn around the strip (*above*) and the strip is cut along the



line, the result, as shown here, is not two strips but a two-sided strip. The mathematicians' explanation: a Möbius strip has but one edge; the cut adds a second edge—and a second side.

A Twisted World of One-sided Surfaces

Topologists enjoy creating odd shapes and strange objects. Among the most curious of these is the one-sided surface, introduced by the German mathematician and astronomer Augustus Ferdinand Möbius (1790-1868). In an article published after his death, Möbius described his remarkable paper surface as a strip which has no "other side." This one-sided strip, hard to imagine but easy to construct (*above*,

left), has all kinds of unexpected properties which are shown on these two pages.

Another German mathematician, Felix Klein (1849-1925), following Möbius' lead, devised a bottle with but one surface—i.e., it has an outside but no inside (*opposite*). Such a bottle, if it could be cut in half lengthwise, would fall into two Möbius strips.

The work of Möbius and Klein has al-

ways fascinated laymen. Some years ago a limerick writer noted: "A mathematician confided / That a Möbius band is one-sided, / And you'll get quite a laugh / If you cut one in half, / For it stays in one piece when divided." Another poet finished the story thus: "A mathematician named Klein / Thought the Möbius band was divine, / Said he 'If you glue / The edges of two, / You'll get a weird bottle like mine.'"



COLORING A MOBIUS STRIP

Anyone can paint an ordinary paper ring red on one side and green on the other. But, as one mathematician said, "Not even Picasso could do that to a Möbius band." If anyone tried, he would only prove (*right*) that the strip has only one side—on which both colors must meet.

Figure 15: "Möbius Strips"

I ran into the problem, well-known now, that taking geometry right between Algebra 1 and Algebra 2 may not be the best approach for algebra *skill development*, which is achieved by not just practice, but spaced repetition, in order to get the skills to move from short-term memory into long-term memory. So I found that my skills had regressed, somewhat, going into Algebra 2. And I did not take Trigonometry — if I recall correctly, it would have been taught by the same teacher that I had for Algebra 2, and I had found him to be a terrible teacher. My high school did not, as far as I can recall, offer anything beyond trigonometry; AP Calculus was not on offer, but even if it had been, I wouldn't have been adequately prepared for it.

My SAT scores wound up extremely lopsided: I had a very high verbal score, but a completely mediocre math score. When my computer teacher asked me my math score, and I told him, he exclaimed “you oughtta be shot!” Because while I was achieving mediocrity in my math education, in the computer lab, I was programming rings around everyone else, spending my class time working on personal projects, like an Apple II BASIC program that allowed the user to define a database, populate it, and save and load these customized databases from floppy disc. And at home, I was learning assembly language, code to generate bit-mapped graphics on a printer, how to write parsers for text adventure games, and other topics that went far beyond the basics of BASIC.

Math and Me — the College Years

And so, when I started college, while my classmates were starting out with AP or IB Calculus credits, I spent my first semester taking remedial college algebra. Which, don't get me wrong, was very helpful, as I belatedly got some of the techniques and rules I was weak on down cold. The class hammered hard on skill-building. But this put me a semester behind my classmates. And then I had to spend another semester taking an Elementary Functions class — a decent enough class, except for the fact that it was held at 8:00 a.m., which made it difficult for me, a deeply committed night owl. While taking that class, I was working a night shift as a computer operator, so some mornings I'd wind up going to class without having slept at all.

I did learn some things in this class, but I didn't build skills as well as I might have, in part, I now believe, it was too *linear* to effectively cement some of the necessary skills into my brain; we'd learn enough of a particular set of skills to complete a section and pass a quiz, and then move on, and might never circle back to reinforce the skills. But I got through it, and then was, at least on paper, ready to take Calculus 111.

Calculus and Me

Some of you might know that back in college, I had a change of plans. I had planned to be a Computer Science major, as I was really interested in programming and had *been* programming since I was about ten years old, but a

few semesters in, I realized that I just wasn't going to manage to get through the required math courses to complete a major. And it was Calculus 111 that made me realize this.

When I write that I "wasn't going to manage to get through" the math classes, please understand that as a student almost entirely funded by scholarships, the bar was higher for me than it was for some of my peers. I had to maintain a high grade point average to keep my funding. It wasn't enough to just *get through* the math courses. I literally couldn't afford to get through them with average grades. And I also couldn't risk not finishing in four years, if I had to retake a class.

It wasn't just two semesters of calculus. There were a lot of math classes lined up between me and a Computer Science major. Completing the requirements for a Computer Science major at the College of Wooster in the late 1980s also met nearly all the requirements for a math minor, and I think some of my classmates completing their major went ahead and completed minors in Mathematics. My memory may be faulty but I think the list included discrete math, differential equations, linear algebra, abstract algebra, and perhaps others I've forgotten — along with Theory of Computation.

From my classmates who had other options, such as taking Calculus over their summer breaks at different institutions, I heard that the Calculus classes at Wooster were very proof-heavy compared to the same classes at other schools, targeted more at math majors who were destined to later take courses in real analysis and other more advanced topics.

All of this weighed heavily on me, taking Calculus 111. I did complete the class, but I found it an unpleasant, stressful experience, for various reasons I am still processing. In particular:

- I often wasn't *satisfied* with the understanding of the topics I was learning. I always wanted to be able to have an intuitive understanding of a concept, which for me usually included a way to *visualize* it. But my classmates in Computer Science would generally say "don't worry about that — you know the rule, just plug and chug!" And I simply *hated* it whenever I had to solve a problem without having clear *insight* into the problem. (I now think that this was an indication that I was thinking more like a budding *mathematician*, and less like an engineer who uses math only as a tool, and that insight often only comes *after* learning mathematical techniques — but at the time it seemed like a profound personal failure.)
- Completing the homework was *demoralizing*. The graders did their best to be kind and not deliberately cruel, and they'd often give me half-points for problems I got almost right, but I was chronically bad at getting through a long series of algebraic steps without making a simple error somewhere along the way, so my homework assignments often resulted in a lot of missed points. I've now come to understand that this was likely due to a combination of insufficient skill development — I had not yet

developed what is called *automaticity* in all of the precursor skills needed to reliably complete my calculus homework problems — and an aspect of my neurodivergence that I call the “imp,” that I still struggle with today, which I’ll go into later.

- Getting through the quizzes and exams was *stressful*. Due to the same factors that led to my poor performance on homework, I could almost never complete the problems fast enough; I’d run out of time. So even when I passed, I lost points for not completing all the problems. My heart would race. I’d break out in a cold sweat. My hands would shake. And those experiences created *trauma*, resulting in *math anxiety*, which further caused my mind to go blank when confronting a timed test, resulting in an even weaker performance.

Almost thirty years later, I remember the math-test trauma at a very basic, physical level, and it *still happens*, although, as I’ll discuss, with regular practice and accommodations, my anxiety has lessened. But I’ve come to understand some things that eighteen-year-old me didn’t yet understand about myself, and that helps.

Recently, I’ve been listening to a podcast called Math Therapy. It’s been helpful to realize just how *many* people found their experience learning math to be humiliating and traumatizing. I have enjoyed listening to these interviews, and it has helped me to re-think what it means to be “good at math” or a “math person.”

I never really planned to go beyond an undergraduate degree, as I just found formal education too stressful, too costly, and too competitive. I’ve benefitted a lot from being a trained writer in a field where an awful lot of software engineers aren’t. I’ve often taken on the role of technical writer, as well as delving deeply into programming. But I can’t deny that I’ve always felt as if I failed at something I should have succeeded at.

Why? Because I’ve always loved math and been interested in math, even as I had certain difficulties with it, difficulties that I *felt*, fundamentally, that I *shouldn’t have had*.

Things I Now Understand about Math and Me

- Getting thrown back into fractions and decimals after successfully learning topics from high school algebra undermined my self-confidence and self-understanding of myself as a “math person.” (The idea that there are “math people” who are *inherently* good at math and if you aren’t one, you’ll never be able to learn math, is pernicious myth with a grain of truth behind it, but I’ll get to that later.)
- The lack of role models in my life who were good at math, and could give me motivation and encouragement in math, was probably a factor. My grandfather, a chemist, who loved to talk science and chemistry and physics with me, died when I was ten years old, and no one really replaced him in

that mentoring role; early on, I became an auto-didact, and it became a defining, immutable aspect of my personality. Even when I had a heavy class load, or a heavy work load, I've always worked on side projects in which I taught myself.

- The lack of good, effective, inspiring math instruction in high school certainly didn't help. I was fortunate enough to have some inspiring teachers in English and History and Chemistry and Physics, but not in math.
- As a so-called "gifted" kid, to use one term that was used to describe me, along with "lazy," I had not ever really developed good study habits, because in most of my classes, I didn't need to. I suspect this is a very common problem; back in the day, I vaguely understood that it was related being an "underachiever."
- Math requires a different sort of learning than other subjects do. I'll go into this more later, but in short, it requires *developing skills* rather than simply memorizing facts or understanding theories, and what study habits I *had* were more geared towards the latter.
- I didn't really need to actively *drive myself* to do the related skill development when it came to programming, as I had already learned quite a bit of programming before entering college, and was highly *internally* motivated to put in the hours to learn more. As a result, I had a hard time understanding how I could be so good at programming, but so bad at math. I think people around me were puzzled by this as well.
- In my fifties I have come to understand that I have always had several neurodivergent traits that might now be called autism and/or attention deficit disorder, and these affected my ability to complete my math class work — in particular, to complete homework, quizzes, and tests. These are traits that still affect me in workplaces and affect my ability to teach myself from standard textbooks.
- In programming, I was able to *work around* the difficulties these traits raised by putting more time into experimenting with, and revising, code, but in the context of timed math quizzes and tests, that wasn't possible.
- I have also come to understand that with some accommodations, these traits might not have been as big a barrier as they were — but I did not receive those accommodations; I don't think it was widely understood that it was possible to both be gifted, academically, and *at the same time* have learning disabilities. Now, I think that this combination is actually common — when one has a brain that is quite different than a typical brain, these differences may manifest as both advantages and disadvantages in academic contexts.

I don't really regret the decision to take an English major and Computer Science minor, as I've always loved literature and writing as much as computer science, and I started off taking English classes as a freshman. And although others might disagree, I don't think my educational decisions really harmed my somewhat irregular career path; it was always destined to be irregular, as I have always

had trouble staying interested and motivated at one job for more than a few years.

But the fact remains that I have never been quite satisfied with my math education, and I've always wanted to go farther.

Math and Me: After College

I have written elsewhere on how, in specializing in software design and development, I became, informally, a kind of engineer without an engineering degree; I've declared in other essays that I am not a "defective electrical engineer," but something different. I've been, generally, able to do enough math to meet my needs as a software developer, including implementing formulas and algorithms as needed.

As part of my paid work over the years, I've written code that implements digital filters, runs PID control loops, calculates standard deviations, smooths and interpolates data, and scales and graphs data. I have rarely found a need to know much more math than I do. But I've always been jealous of the electrical engineers I've worked with who use math as a tool with much more *facility* than I do — especially Ph.D. engineers who can cover a whiteboard with the math behind a digital filter algorithm.

And beyond that, I've always wanted to learn a lot more math for *fun*. I love reading books and articles in physics and electronics and computer science. I've tried many times over the years to be my own teacher and work on math. I used to collect all the textbooks I could on calculus, analysis, topology, Euclidean non-Euclidean geometry, number theory, set theory, lambda calculus, proofs, formal logic, electronics, cellular automata, parsing, compiler and interpreter design, type theory, and algorithms.

I have gotten rid of many of these over the years — I no longer have half a dozen books on linear algebra, for example — but I still have quite a few, including a copy of the same calculus textook I used in Calculus 111, decades ago, and several others. I also have collected a number of books on so-called "recreational mathematics" — books by Martin Gardner, Raymond Smullyan, A. K. Dewdney, and others, and I still enjoy solving logic and math puzzles, such as the Cheryl's Birthday puzzle I've discussed in the past.

I *have* been able to keep a little bit of practice going over the years. I have worked my way partway through textbooks on linear algebra, category theory and other topics. I've learned a bit this way, but have never fully completed one of these textbooks or felt like I have fully mastered the topic. I now know that this is mainly because working through the textbook on my own was not sufficient for mastering the topic; even if I understood the "lecture" portions, I hadn't really done the *skill development* that I needed to do in order to achieve full mastery of, say, linear algebra.

But I do often have the *insight*, when looking at problems or puzzles. The sketch

of a proof or solution pops into my head. Sometimes I can make it work, and sometimes I get stuck because I get hung up on not knowing how to take a step correctly. It's frustrating to often find myself *thinking like a mathematician*, without consistently being able to *do the work*.

My frustration has been compounded by my desire to homeschool my kids in math, which has been very difficult to do while also holding down full-time (or more-than-full-time) jobs. So I've tried to find ways to do this without being able to put in much time.

For a while, I made a concerted attempt to put them each in Khan Academy courses. They were for the most part totally resistant to finishing their assigned work, although it was "gamified." I also tried working through some Khan Academy courses myself, as a refresher. I could do the work, but I didn't feel that I was learning the subjects very well. I didn't actually like the video lecture format, and the "gamification," and the coverage always seemed to leave things out, so I'd wind up feeling like I finished a video lecture with more questions than answers.

A few years ago, I hired a math tutor to work with the kids, and a couple of them got some value out of that — Sam, in particular, worked his way through some great material on probability and statistics. I would have kept that going, but unfortunately the tutor had a change in life circumstances, and was no longer able to work with us. I had been considering hiring a tutor to work on the math that *I* wanted to learn, but seven kids at home and a full-time job spells "very unpredictable, chaotic schedule." It is hard to determine in advance when I'll be free to meet with a tutor, or free to work on homework. That's one advantage to working with an online program — you can do it in whatever scraps of time you can find, although unfortunately on some days, that may be "none at all."

Math Academy

About eight months ago I read a post on Hacker News, which led to a blog post that described a startup called Math Academy, which was looking for beta users for their online math education system. I was skeptical that it was another Khan Academy. But as I looked into it, I found that there was some good pedagogy behind the site.

The aspects of the program that particularly appealed to me were as follows:

- The user interface is simple to use, but shows "quiet sophistication." It is low on distractions and immediately visible features, but it lets me do everything I want to. As someone who once had a job designing instructional multimedia, I can recognize the thoughtfulness in this design.
- It is not really "gamified." There are "leagues" where you can rank yourself against other users, competing for experience points completed, *if* you find that to be a helpful motivator. I used it for a while, but it was mostly reminding me that I don't get as much time to work on math as a full-time

student, so I turned it off completely.

- It is designed to instill *automaticity*. It will hammer you with reviews that are spaced out over time in order to maximize retention. It will frequently “circle back” and give you review questions.
- It periodically throws “multistep” problems at you — a “multistep” is a series of problems that follow on from a real-world context, and build on each other.
- It gives you quizzes so often — roughly weekly — that eventually they will become routine. Early on, these timed quizzes would bring on the same mild sense of panic that I recall from college math classes — the sweating, the racing heart, the trembling hands. But my responses to them has become gradually calmer over time, as I’ll discuss further below.
- The quizzes are *low stakes*, something that was never true in Calculus 111.. The worst-case scenario: you’ll have to review some topics, or you’ll have to retake the quiz (with different problems). The goal of the system is mastery, not just passing.
- Despite being *low stakes*, some of the quiz questions are quite difficult — they will sometimes require putting together multiple techniques in a way that you haven’t been shown before. And — I just learned this — the algorithms choose quiz questions calibrated to your progress. If you are consistently scoring above 80%, it will make the quiz questions harder. This means two things: one, it’s normal and expected to miss a couple of questions, and two, the fact that the quiz questions are getting harder is a good sign. When I get a quiz problem marked “H” (for “hard”) correct, I feel like a frickin’ genius!
- The topics are *comprehensive* — they go into a great deal more detail than other online learning resources (and even most textbooks) go into.

Healing Math Trauma with Accommodations and Low-Stakes Testing

To try to heal some of my math trauma, I’m using an accommodation setting, which allows extra time for quizzes. I’ve told it to give me additional time to complete quizzes. You can tell it to give you 25%, 50%, 100%, 150%, or 200% additional time. For example, if it would normally give you ten minutes to complete a quiz, the 200% setting would give you thirty minutes.

Initially, I set it to 200%, which did wonders to take some of the pressure off. It would give me the ability to go back over problems and double-check my work. And, gradually, I have found myself responding to quizzes with less intense physical symptoms. I’m currently got the system set to give me 150% additional time, and I now usually complete a quiz with ten minutes or more left on the clock. I may wind up lowering it further, as I don’t seem to actually *need* all the extra time for most quizzes, but for now, the fact that I *could* take more time seems to be very significant, and it helps keep me from feeling that old familiar panic, when my mind goes blank and I forget everything I’ve learned.

But despite this improvement, and lowered anxiety, there’s still an imp in my

brain, and I think it's always been there.

Working Around the Imp in My Brain Having filled seven notebooks so far:

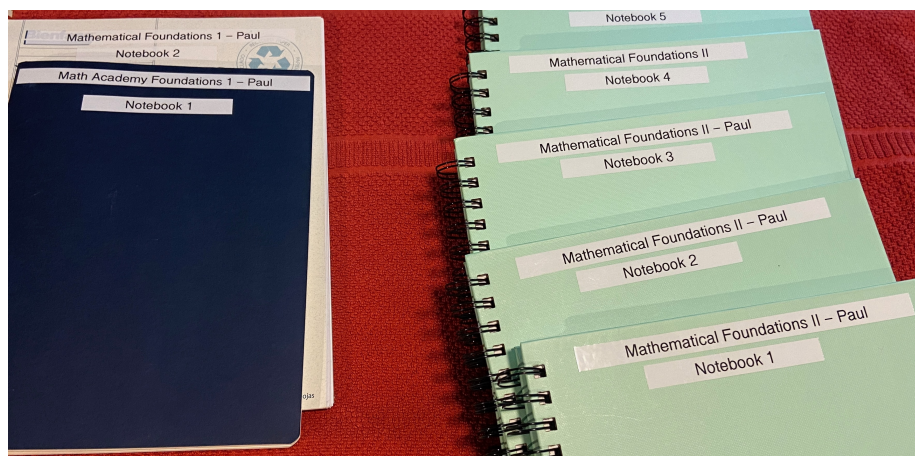


Figure 16: “My First Seven Math Academy Notebooks”

I've been able to collect a lot of evidence of just how the imp in my brain works. I've been taking mental notes, and sometimes paper notes, about the kinds of *errors* I make. The study of errors seems to me like it must be a very important way to gain insights into the different ways that skill-building is working, as well as not working.

The imp seems to be constantly trying to get me to make mistakes. I describe it that way because that's what it feels like. I can be thinking (and hearing) the correct next step in solving a problem in my head, but when my hand actually writes it down, it often seems that the imp has short-circuited the messages my brain has sent to my hand, and I've written down something wrong. It's almost like I had a kind of micro-seizure — and it's been happening to me since childhood. Going back as far as I can remember, I always have made these kinds of errors, despite knowing and reciting to myself the correct answer.

The imp will drop a minus sign, even though I just heard my inner voice say the expression *with* the minus sign. I'll hear myself say “2,” and I'll glance down and notice that the imp has written down “t” instead of “2.” I'll be looking at an expression on the screen like “ $p(x) = (x - 15) * q(x)$,” and I'll start writing it down, but then glance at the paper and realize that what I started to write was “ $p(x) = (q - 15)$.” I'll be in the process of going through several steps of algebraic manipulation, and feeling confident that I'm doing it right, and *hear* in my head, as clearly as I hear other people speaking to me in the so-called “real world” what I should write down next, and I'll write it down *wrong*.

I make mistakes like this *constantly*, no matter how hard I try not to. Many times, it seems like the errors happen because my brain has moved on to the next step in the problem and raced ahead, before my hand has caught up, and will give my hand superseding orders while my hand is still carrying out the previous orders. So I'll start writing down a term from the *next* step before finishing the previous one.

It seems almost like the way my brain short-circuits sometimes while I'm eating, and at the same time thinking about too many other things, and as a result the moving parts in my mouth get out of sync, one of those moving parts gets an incorrect order, and I wind up painfully biting my own tongue, or my cheek. Except that it happens a lot more often.

In the context of classroom work, or online learning, my great difficulty with the imp is that it causes me to make errors which suggest I don't understand the problem and the techniques to solve it, or that I am guessing, which is not true. And so I often wind up losing points (in a graded academic setting) or given review work I didn't truly need.

I can *work around* the imp somewhat, by slowly and carefully re-reading each step in my calculation and correcting any errors that I find that the imp has caused. But this takes extra time, and sometimes the errors are hard to notice.

I think *that*, more than anything else, is the reason that timed quizzes and tests have always been a source of extra stress — because the time allotted is allotted under the assumption that the student isn't locked in a constant struggle with his or her imp.

Oddly, when I'm very tired — as in, when my eyelids are starting to close involuntarily, and I'm on the verge of falling asleep from exhaustion — it seems that I never make this kind of mistake, and I get 100% on my quizzes. Maybe the imp has fallen asleep? I've reproduced this effect several times, but as a strategy goes, it's hard to put into practice, as I don't actually want to go with too little sleep.

This technique — working while very tired — actually helps, in a way, with my regular assigned problems. I can get the problems correct, with fewer imp attacks, but it takes me much longer to complete the problems, as when I'm tired, my ability to reason through the solutions is slowed down.

Take this explanation with as many grains of salt as you like — after all, it is my *subjective* experience. I've never done math problems while my brain was wired up, or while I was in some sort of scanner. Subjectivity is all I've got. But I will try to study it further. I already write out any quiz problems I got wrong, correctly, but maybe I should also start keeping an “error journal,” in which I document every error that I discover the imp has made me make. Maybe it will give me some insight into what it wants and how to get it to leave me alone.

Maybe the answer is that the imp is really a hard-wired, neurological manifestation of my impatience and my wish to learn faster, but if so, it is not helping

me achieve that wish. Maybe it is a signal that the other hemisphere of my brain wants more attention and stimulation, and isn't getting it, and is trying to sabotage what I'm doing in order to get it. I don't know!

Working Around My Aging Eyes There's another challenge I face that is different than the imp. That challenge is simply that not only are my eyes not great, and continuing to age, but as I work on screen all day for my day job, by the time I get to my Math Academy work, they are often very fatigued, as well. So I sometimes have to zoom way in on the screen to resolve the difference between x and z, notice signs, and read exponents and subscripts. And I have a bit of the same difficulty reading my own writing. But that's a more *understandable* and *predictable* problem, and so easier to work around than the imp.

The Mathematical Foundations Series Anyway. To start a class in Math Academy, once you've signed up and paid them, you can assign it to yourself, or if you have set up accounts for your students, you can assign the students a class. The first thing any student has to do, when starting a class, is to complete an un-timed placement test. This will then lead to a test report, which includes advice that you are free to either accept or ignore. If you do very well, it might suggest that you try a more advanced class. If you do very poorly, it might suggest that you start with a less advanced class. Or, it might say you're in the right class.

Trying to be realistic about how much I thought I remembered from high school and college math, I initially placed myself into a class called Mathematical Foundations I, taking the placement test on April 12th. The system told me that I had guessed correctly and Foundations I was a good starting point for me.

The Foundations series, I through III, are large classes designed specifically for adult learners who need some combination of review of material that you once knew, and new material to fill in gaps. I started Foundations I at about the 70% mark. It turns out that I still remembered a lot of algebra, including how to handle exponents and roots (because of all the exercises I did in remedial algebra, my freshman year in college), but I had forgotten some techniques involving inequalities (which I rarely used in college), and a number of postulates from geometry, which I also hadn't used since high school. So it was a good refresher.

I finished Foundations I on May 23rd. There is no great ceremony, currently — the system just rolled me over into Foundations II.

Taken together, Foundations I to III covers material usually covered in the following classes (roughly speaking; they typically go into more detail, covering more techniques and topics, than the classes you are likely to have taken in the past):

- Junior High School Prealgebra
- High School Algebra I

- High School Geometry
- High School Algebra II
- High School Precalculus
- High School AP Calculus AB *or* College Calculus I
- High School AP Calculus BC *or* College Calculus II

Seven courses, roughly, but in great depth, and — this part is very interesting — worked through *in parallel* as much as possible, advancing through topics *in small steps*, and constantly circling back to review and re-review topics you’ve previously passed, as part of that effort to make it so you learn *skills* and get them *down cold* to the point of *automaticity*.

For reference, here’s Math Academy’s page describing their courses, showing the different ways a student can work through them. And here’s an interesting Twitter thread from the guy that designed the series.

Currently, I’m getting close to the halfway point of Foundations II. I’m covered some trigonometry, a lot of material on functions, and some calculus. By the end of this course I should have covered some of what I learned in Calculus 111, along with a lot of other material; here’s a comprehensive breakdown.

How I’m Working Since starting my work with Math Academy, I’ve been renewing my love of mechanical pencils. It turns out that while the basic technology is the same, there are new kinds of mechanical pencils that I’ve never used before! So, I am now working with 0.2mm and 0.3mm Pentel Orenz pencils purchased from JetPens. These have the finest lead I’ve ever used. The pencils have a little steel sleeve that actually touches the paper and slides into the pencil as the lead wears down. This sleeve keeps the lead from breaking, which otherwise would happen constantly given how thin and fragile 0.2mm and 0.3mm lead is. This fine lead allows me to print legibly even when the letters and numbers are very small, and draw fine details on graphs. I order B hardness lead, which is a little darker than HB, and less likely to cut into (and snag on) the paper.

I also have in my pencil case several different sizes and shapes of erasers, including this Tombow Mono Stick Eraser, this narrow-tip eraser, and this even finer round-tip eraser. These are terrific products that I recommend. I don’t think erasers like this existed the last time I was doing math homework. They are great for doing precise touch-up without having to erase whole lines of work. I often use the smaller-tipped erasers to touch up drawings of graphs.

For notebooks, I have tried a few different types, and settled on these hardcover spiral-bound notebooks from Michael’s which I’ve ordered by the 12-pack. The paper has a reasonably smooth texture. I’d like a smoother paper, but these are only five bucks each when purchased in bulk, and I can’t resist that.

The kids prefer different notebooks, so I’m keeping a stock of several different kinds on hand for them; Sam likes dot grid notebooks, and Joshua likes notebooks

that have lines on one half the page and blank space on the other half. They are nice for problems that require you to draw graphs or number lines as well as calculate.

So what do I write down? I actually write out the complete lessons, in condensed form, as well as all the practice problems, with my solutions. When I started Foundations I, I found that my handwriting (really, printing) had deteriorated considerably. This motivated me to try to improve it. My print now looks very close to how it looked when I was in college, although my hand becomes fatigued faster than it did.

When the system checks my work, even if I got the problem right, if the online system shows a more efficient way to get to the same answer, I'll write out the example solution in full, too, to study the recommended approach. I've learned some useful shortcuts this way. My typical approach has been to *expand out* problems completely before starting to factor and simplify, but the example solutions often show ways to cancel and simplify expressions as early as possible, which results in having to write out fewer terms and take fewer algebraic steps, leading to a lower likelihood of errors.

Here's a geometry problem involving, ummm, cutting a cheese, showing my solution along with the Math Academy web site's solution:

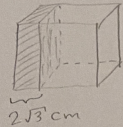
I got the correct answer, and I like the second part of my solution, but using the reciprocal to find the remaining volume is more elegant. I thought about doing that, but as this is one of the problems I did while I was very tired, I decided to do it in a way that seemed, to my tired brain, more straightforward.

The creators of Math Academy don't actually *advocate* writing the full lessons down the way I do, although they *do* advocate always writing out your problem-solving steps, and advise you to avoid, as much as is reasonable, skipping steps that you can do in your head. But I always used to write down lectures like this in classes — it helped me concentrate on the lectures. I find that it helps me, *even if I never go back to review what I've written.*

Also, I think that I'm fundamentally a bit uncomfortable in paying for a class and not winding up with a printed textbook, so writing the course material down makes me more comfortable, even though I am working through it in a highly personalized order, jumping between topics as the algorithm shows them to me. I carefully erase mistakes and try to make the notebooks look as neat as I can, within reason — I'm drawing everything freehand where I could theoretically use a ruler or a protractor, but I can't bring myself to be quite *that* neat.

Personally I think the Math Academy folks *ought* to sell review workbooks. The founders would probably say that having a workbook full of topics in a fixed order, that wasn't tailored for the user based on performance and quiz results, ignores the whole point of the highly-tailored experience. But I still think it would be cool to have workbooks that take transverse slice through topics. For example, I would definitely buy a paper workbook that was all about factoring

- 8) The diagram shows a cube of cheese. Paul cuts off $\frac{2}{7}$ ths to use. Find the volume of the remaining cheese.



Note that to find the original width and height we can

$$\text{Use } \frac{2}{7}h = 2\sqrt{3}$$

$$\frac{1}{7}h = \sqrt{3}$$

$$h = 7\sqrt{3}$$

So the original volume was

$$(7\sqrt{3})^3 = 343 \cdot 3\sqrt{3} \text{ cm}^3 \\ = 1029\sqrt{3} \text{ cm}^3$$

Cut off $\frac{2}{7}$ ths:

$$\text{And } 1029\sqrt{3} - \frac{2}{7}(1029\sqrt{3}) \\ = 1029\sqrt{3} - 294\sqrt{3} \\ = 735\sqrt{3} \checkmark$$

Note: Textbook solution:

Since $2\sqrt{3}$ is $\frac{2}{7}$ of any side's length,
the original length of any side is $\frac{7}{2} \cdot 2\sqrt{3} = 7\sqrt{3}$
(easier to solve)

The remaining volume forms a rectangular prism
with volume

$$7\sqrt{3} \cdot 7\sqrt{3} \cdot 5\sqrt{3} \\ = 245\sqrt{27} = 245 \cdot 3\sqrt{3} = 735\sqrt{3} \text{ cm}^3.$$

There is $735\sqrt{3} \text{ cm}^3$ of cheese remaining.

At 46% of foundations II

Figure 17: "A Cheese Cutting Problem"

techniques, with tons of sample expressions to factor, and an answer key, graded from the most basic to the most complex. I would work through that workbook and I think other students might enjoy doing the same.

What I don't write down in the notebooks are quiz questions. For quizzes, I use Blackwing Matte wooden pencils, which allow me to write faster, and I try not to erase anything, as that slows me down — if I've made an obvious misstep, I just scratch it out and try again. I do the quiz problems in different notebooks and I don't put any effort into making them look nice. But when I get a question wrong on the quiz, I write the correct procedure out in my main notebook, to try to reinforce learning how to do it correctly. Over time, the number of problems I have to write out in this way is getting smaller and smaller.

Initially I was creating crib sheets and index cards with various rules and formulas. This is discouraged, though — the idea is that you want to actually memorize the formulas you need to memorize, and using a crib sheet can slow down this process rather than accelerate it. So I've been doing that less and less as well, as I gain confidence.

What I've Learned So far, I've re-learned many topics that I know I have learned before, sometimes in greater depth than I ever learned the topics before, but along the way I have also learned some topics that I don't recall ever being exposed to in my previous formal math education, including:

- Synthetic division of polynomials by binomials
- Long division of polynomials
- Some techniques for solving equations involving logarithms (we probably covered these in the functions class in college, but if so they didn't stick very well)
- Finding the numbers of permutations and combinations using factorials
- Calculating variance and standard deviation
- Arithmetic, recursive, and geometric series (I knew a little about series from studying recursive algorithms, but Math Academy goes into quite a bit more detail)
- Imaginary and complex numbers including imaginary roots, and math on complex numbers
- Several new-to-me factoring techniques for working with cubic and quartic polynomials
- The rational roots theorem
- Heuristics for graphing cubic equations and other polynomials that I don't recall ever learning (although, again, we might have gone over it my long-ago functions class)

My Goals I'm hoping to finish Foundations II by February or March of 2025, and then complete Foundations III, which, assuming things go well, I might be able to complete by the end of 2025. This is based on putting in only fifteen or

twenty experience points a weekday, which for me translates to something like thirty minutes to an hour of work, although on some days I am able to do more.

Interestingly, I have found that for the purposes of *learning skills*, it is often better to do *less*. On days when I had a little extra time, I would push myself through three or four topics, because I was enjoying myself. But I gradually found that the work I did on the third or fourth topic was nearly useless, as far as retention goes; I would not be able to remember those topics when given questions about them on the next quiz. It was more effective to complete one topic and a short review or two, and then stop for the day.

I have some confidence that, given what I've managed to achieve both back in college and recently, I probably *can* complete Foundations III. I think it will, *approximately*, get me to the point where I would have gotten, if I had started college better-prepared, and been able to make it through Calculus 111 and Calculus 112. In other words, getting through Foundations III seems to be *a place it is possible for me to climb to*, given where I am standing now.

Then, the question becomes, in part, *what can I actually achieve, in mathematics*, over the course of the rest of my life? (There are, of course, no guarantees, but let's imagine that I've got ten more years of reasonably good brain function.) I would like to achieve quite a bit:

- I would like to complete a Math Academy course on proof techniques.
- I would like to complete some additional courses; some, like discrete math, or probability and statistics, might be relatively easy after getting through Foundations III. Some, like multivariate calculus, might be relatively hard.
- I would like to get to the point where I can complete at least the first volume of Terence Tao's textbooks, *Analysis I* and *Analysis II*, Third Edition. Why that book? For one thing, I've got both volumes on my shelf. For another, I like Tao's writing style and his gradual approach. The first volume does a great job of informally and formally building up the "rules" of mathematics, and I'd love to be able to understand the basic mathematical operations on the natural numbers, on the integers, on the rationals, on the reals, and on complex numbers from this perspective. It would be nice to get through the second volume, and it probably looks quite a bit easier from the perspective of having mastered the first volume. But from where I'm standing now, I'd really be quite overjoyed just to master the first.
- I would like to be able to work through a number of other books I've got, including books on digital signal processing, electronics, physics, and other topics — even if they take me a long time. If I can afford the time and money, I might be willing to work with a tutor to reach this goal.

And, of course, I'd like to be able to continue to homeschool any of my kids in math, continuing into more advanced topics, if I can stay ahead of them. If not — maybe they can homeschool *me*.

The question of of just how far a motivated older individual, gifted in some ways,

but perhaps with some learning difficulties, might be able to go in mathematics, is not a simple one. But two of the guys behind Math Academy, Justin Skykac and Jason Roberts, have a book in progress. In this book I've found the best answer I've ever read to this question, once you get past two obvious wrong answers: "the sky's the limit!" or "I'm not a math person."

Interestingly, this answer comes in the form of comments by Douglas Hofstadter, someone I always admired, going back to my teenage years, when I first read his book *Gödel, Escher, Bach: An Eternal Golden Braid*. In fact, I've always hoped to be the kind of thinker that he is.

Hofstadter writes:

I am a 'mathematical person,' that's for sure, having grown up profoundly in love with math and having thought about things mathematical for essentially all of my life (all the way up to today), but in my early twenties there came a point where I suddenly realized that I simply was incapable of thinking clearly at a sufficiently abstract level to be able to make major contributions to contemporary mathematics.

This might seem like an eye-rollingly obvious comment to most people reading — anyone who didn't make it through high school algebra might not be very sympathetic to the pain of *seeing* and *recognizing* mathematical genius, and realizing that one is not going to *be* one — but I really feel it. As someone who was always told how bright I was, by authority figures I trusted, maybe it shouldn't be a great surprise that I believed them. And I was told "the sky's the limit," which... is less true. It's the difference between the ambition of one's *loves* and *interests*, and the reality of one's abilities.

Hofstadter continues:

I had never suspected for an instant that there was such a thing as an "abstraction ceiling" in my head. I always took it for granted that my ability to absorb abstract ideas in math would continue to increase as I acquired more knowledge and more experience with math, just as it had in high school and in college.

I really feel this, too. I know, from having worked as a software engineer for decades, that my "abstraction ceiling" is — or at least, *was*, given how aging and COVID have slowed my brain — much higher than that of most people. In fact, of the hundred or so co-workers I can recall working with, I can only think of two or three who were capable of designing and building not just programs, but whole software *systems* at the level of abstraction that I used as a matter of course. And I have always learned new programming languages for fun. I've never felt like I've really reached the *top* of my programming ability, or what I had the *ability* to understand (although trying to understand some highly abstracted Haskell programs definitely causes me to hit the limits of my current understanding).

But this does not necessarily translate into having the abstraction power that some of the mathematicians whose work I read and admire so much, like John Baez, Leonard Susskind, or Roger Penrose, actually use in their work.

Hofstadter continues:

I found out a couple of years later, when I was in math graduate school, that I simply was not able to absorb ideas that were crucial for becoming a high-quality professional mathematician. Or rather, if I was able to absorb them, it was only at a snail's pace, and even then, my understanding was always blurry and vague, and I constantly had to go back and review and refresh my feeble understandings. Things at that rarefied level of abstraction... simply didn't stick in my head in the same way that the more concrete topics in undergraduate math had... It was like being very high on a mountain where the atmosphere grows so thin that one suddenly is having trouble breathing and even walking any longer.

Skycak and Roberts raise the fact that the difficulty level of mathematics doesn't necessarily increase, as one climbs the "mountain," in the way that one thinks it might. It isn't *linear*. They briefly discuss how

Most people consider calculus to be "really advanced math," but calculus is not even halfway to the level at which expert mathematicians operate.

I don't consider calculus to be "really advanced math." I consider it to be the point where the student learns a *toolkit* of techniques that start to be useful for solving real problems, and also the point where the problems can start to become truly abstract. Then, *analysis* is where you really start to learn the answers to the *why* and *when* and *how* questions about the tools in the toolkit — which is why I want to learn at least some analysis. But past that? There's a *lot* more, and it gets more and more specialized, and the air up there gets more and more rarefied.

Skycak and Roberts continue:

For reference, we offer a loose formulation of the levels of mathematics below:

1. Arithmetic — seldom considered "hard math"
2. Algebra — often considered "hard math" by people who disliked math in school
3. Calculus — considered "hard math" by the general public
4. Real Analysis, Abstract Algebra, Partial Differential Equations, etc. — considered "hard math" by most college students majoring in math
5. Algebraic Topology, Differential Geometry, etc. — considered "hard math" by most graduate students doing Ph.D.s in math, as well as many research professors in math

6. The math underlying solutions to the most famous problems in modern mathematics, e.g. Ricci Flow with Surgery which underlies the proof of Poincaré Conjecture — considered “hard math” by the world’s top mathematicians.

Ascending between those levels isn’t a linear process. Although, Math Academy arranges the topics in such nice graded steps that it feels *locally* linear, at least where I am right now, it gets *steeper* — slowly at first:

As Hofstadter describes, the abstraction ceiling is not a “hard” threshold, a level at which one is suddenly incapable of learning math, but rather a “soft” threshold, a level at which the amount of time and effort required to learn math begins to skyrocket until learning more advanced math is effectively no longer a productive use of one’s time... The central insight is that the further you go in math, the more energy it requires to learn the next level up. Whether they realize it or not, everybody who learns math is on an exponential curve of energy (time and effort) versus the level of math. (A key feature of exponential curves is that they can look fairly flat at the beginning, but appear to skyrocket later on, despite there being a constant “multiplier” to get from one point to the next.)

I think that’s true. I’ve long understood that I wasn’t ever likely to master all the math that Roger Penrose describes briefly in his book *The Road to Reality*, which is probably the Mt. Everest of the physics books I’ve got on my shelf, as far as mathematical abstraction goes. I do still have hopes of getting through the first *Theoretical Minimum* book on classical mechanics, which takes a much more gentle approach, and I’d love to go further in that series. And I think that I really ought be able to get through *Conceptual Mathematics: A First Introduction to Categories* by Lawvere and Schanuel, as they have taught at least some of that material to high school students. And I have a lot more books on my shelf that I’d like to be able to progress further in, especially the ones that are applicable to Computer Science.

The “math gets exponentially harder” concept feels true, but hearing it described that way makes me feel profoundly sad. In some ways, I feel sadder contemplating the idea that there is a finite “ceiling” to what I can master, than I do contemplating that my life itself is finite. Because abstraction has always been extremely important to me. Deep down, I’ve always been someone who loves ideas more than anything else, and abstract ideas more than any others.

I think maybe I’ve been secretly harboring the idea that there’s a sort of Omega Point to my understanding, and if I work hard enough, my mind will be able to, as in Tipler’s interpretation of the omega point concept, go asymptotic, and understand an *infinite* amount in *finite* time. And it’s a little sad to have to give that idea up. (If, in fact, that ever happens, it will happen through a kind of epistemology that I can’t understand now, to a different *me* that I can’t understand now, so let’s set that possibility aside for another discussion — more

of a *theological* discussion — in another spacetime context.)

My Ceiling So, what, *exactly*, is the limiting factor that puts a “ceiling” on the level of math I’ll be able to learn? Is my “imp” the limiting factor? No, I don’t think so, although I think now that the imp *harmed* my progress early on, and accommodations early on would have helped a great deal. Now, the imp slows me down, and I think it’s a quirk in my neurology that requires accommodation, but is for the most part something that I can work around.

Skycak and Roberts suggest it might have a lot to do with *working memory capacity* and that *this* is what puts a “ceiling” on abstraction ability. And I recall in my early twenties, being able to hold whole programs and frameworks in my head, and write code as fast as I could type, which was very fast. I remember wistfully being told by a co-worker at the University of Michigan that she couldn’t believe I was so good at “this stuff,” but so bad at things like communicating with co-workers and dealing with office politics.

I can’t always be “so good” software design and implementation any more, at least not the way I used to. I compensate for it by knowing when to write things down and knowing where to look things up — but it’s not the same. Although, in the workplace, I am finding that experience actually does count for a lot — I often know, now, what kinds of programming and debugging problems *not* to bash my head against out of sheer stubborn will and disregard for the effort an approach will take, but instead, how to find a better way. And I’ve gotten far better at working with co-workers.

All that said, I’m quite confident that there’s more I can learn before I hit the point where it’s all diminishing returns. But — maybe not *much* more.

I’ll never get back the brain I had in my late teens through my early thirties. It was a remarkable thing. I probably should have taken better care of it, and *definitely* shouldn’t have fed it so much beer that one time, or that other time, and probably would have been better off if I hadn’t fallen down that flight of stairs as a toddler that one time, which is why I still have that weird bump on head to this day, or gotten that concussion doing a dive roll in tai chi class, or gotten beaten up in school that one time... or those other times...

Working with the Kids Math Academy is still in beta and there are a few rough edges, but I’ve been impressed enough that after working with it myself for a while, I signed up the three oldest boys and had *them* take pretests. Veronica opted out.

I put Sam in Foundations I, as he’s had enough of an unconventional math education to be ready to take a course designed for adult learners. Joshua and Pippin are in earlier classes. I’ve allowed them to set the pace and decide what their daily experience point targets should be. And although there were some initial frustrations with kids refusing to do their work, all three of them have now made enough progress in the class that they’ve crossed the threshold and become

self-motivated, which is terrific. And they continue to make rapid progress. The plan is to get both Joshua and Pippin through the Integrated Math sequence, which covers all of traditional high school math, but structured so that they don't leave algebra for a semester while taking geometry, and then try to pick it up where they left off — all that material is taught in parallel, in small steps, with plenty of “circling back” for review, to help them achieve automaticity.

Getting through this high-school level material is what Grace and I require and expect of them, but after this? They'll have the choice to continue in math if they choose, or not. Or maybe they'll continue with it in the context of a community college class. It'll be up to them.

Our Seminar Class To help keep the kids (and myself) on track, I've been holding a daily math work session/seminar in the library. After my work day ends, I run downstairs to join the class — which makes my day even longer, but I get a little energy boost from eating a few squares of dark chocolate, and also from the fact that I'm doing something fun.

To facilitate our classes we also got a big whiteboard on a tripod, and a whole bunch of markers:

(Whoops, the captions “ $y = 1$ ” at the bottom should actually read “ $x = -1$.” That might have been the imp at work.)

We have three iPads — the cheapest models available, 10.2-inch 9th Generation devices — that are reserved for the boys to use for math work. To enforce this, our Wi-Fi router limits the hours in which they can connect to the Internet. When they aren't in use, they're in a special charging case in my bedroom, a Copernicus Tech Tub 2 that I got on eBay. That setup looks like this:

Note that one of the five we keep in there is in use, hence the dangling cable.

The three older boys and I do at least some of our daily Math Academy work, accruing our daily “experience points,” during the first half-hour of the class.

If they are stuck on a problem, we work through it on the whiteboard. If they get through their work without getting stuck, which is usually the case, I ask each of the boys to show us a problem they recently were assigned, and they demonstrate the solution on the whiteboard. Then, I will generally either show an interesting problem that I have come across, or a puzzle from one of my recreational math books, or give a short lecture/demo on a math topic. Sometimes they are about things the kids are learning now, and sometimes I am giving them a preview of things they might learn in the future. Recent topics have included:

- Various problem-solving situations in which it is useful to flip fractions upside-down
- Why dividing a number by zero doesn't just equal zero (explained at several levels, from a basic analogy with dividing up a pizza, through an

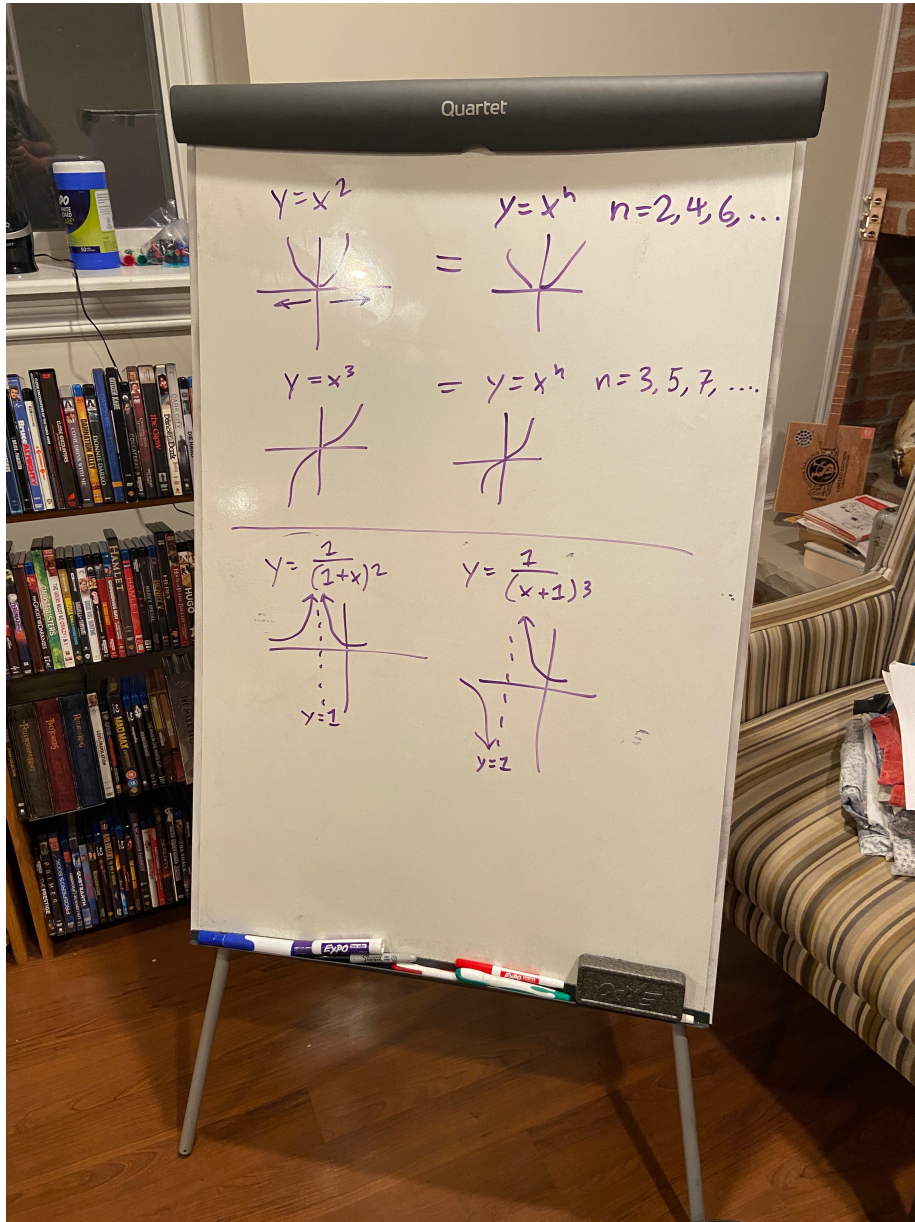


Figure 18: "Our Basement Whiteboard"



Figure 19: “Our iPad Charging Station”

explanation using limits, to the more advanced explanation of how the rationals are derived from the natural numbers)

- How to do both synthetic division and long division of polynomials (this is something I don't think I ever learned, back in the day, but which has been incredibly useful for factoring)
- How to graph different kinds of functions
- Introductions to imaginary and complex numbers
- Algebra tricks like “unwinding” logarithmic expressions using exponentiation, and solving quartic equations as quadratics

Some of these topics are well *ahead* of where the kids are now, but I think that's OK. I'm trying to *expose* them to some more advanced topics which they won't fully learn or understand now, *without* asking them to fully understand what I'm describing. I'm hoping that these little previews of future topics might inspire them, and it might help if the topic looks a little bit familiar when they get there.

The Science Behind Math Academy You can find an overview page on Math Academy here. There's also a on how their AI (not chatbot-style AI, but expert system AI) works, here. If you're interested in the cognitive science and pedagogy behind Math Academy, I'd recommend starting there.

As I mentioned above, Justin Skykac and Jason Roberts, two of the people who made Math Academy, have a book in progress called *The Math Academy Way: Using the Power of Science to Supercharge Student Learning*. This contains a lot more detail. It's fascinating. The two authors are also on Twitter, have podcasts, etc. And there is a Math Academy Discord for students. I'm on it, but I haven't had time to do more than have a few very brief conversations.

In Conclusion, Math is a Land of Contrasts

Bye for now!

You can find all these newsletters in my writing archive.

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